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Review

Solving the problem of electromagnetic wave diffraction at a finite plane grating with small elements

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Two approaches for solving the three-dimensional problem of wave diffraction at a finite grating consisting of bodies of revolution are proposed. An approximate solution is obtained for a grating with small elements. This solution is applied to consider gratings with a large number of elements. The coincidence of the results obtained by the two methods is shown. The reflection and transmission coefficients are compared for finite and infinite gratings.

Contents

1. Introduction

The problem of wave diffraction at particles which are small compared with the wavelength is of great interest in optics, radio astronomy, hydrometeorology (see, for example, [1–4]), etc. There is a wide range of methods, such as the method of current integral equations, the method of T-matrices, and some others, which can be used to solve this problem. Recently, an approach based on the electrostatic approximation [5,6] began to develop actively. We believe that the pattern equation method (PEM), which was proposed in 1992 and has been actively developed since that time (see [7]), is the most adequate method for solving such problems. The fact is that PEM requires the body pattern which, in contrast to the distribution of sources (currents) on the scatterer surface, weakly depends (in the case of diffraction at a body whose dimensions are small compared with the wavelength) on the scatterer shape and the presence of other bodies near the scatterer. PEM permits solving the corresponding problem in the so-called "single-mode approximation" (see below), where the pattern of a single body can be written as the sum of the minimum possible number (namely, six) of terms. PEM also allows one effectively to solve the problems of wave diffraction not only at small

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independent particles but also at groups of bodies, even if such bodies are located close to each other. We note that, in this case, the whole number of unknown terms is equal to $6N_\Sigma$, where N_Σ is the number of particles. Thus the quantity of basis functions is rather small. In this paper, the problem of electromagnetic wave diffraction at a finite lattice consisting of bodies of revolution, which are small compared with the wavelength, is considered. The problem is solved in two ways – by the pattern equation method based on the use of the single-mode approximation mentioned above and by the modified method of discrete sources (MMDS) [7] which, in principle, permits solving the problem with the required accuracy.

2. Statement of the problem and its solution by PEM

Consider the problem of scattering of the primary monochromatic electromagnetic field $\vec{E}^0$, $\vec{H}^0$ at a finite grating consisting of regularly spaced identical bodies of revolution. Suppose that the "centers" of the bodies are located at one plane with the periods d_x and d_y (see Fig. 1). The axes of the grating elements are parallel to each other. Denote the surfaces of the grating elements by S_{jl}, where $j=\overline{-N_1,N_1}$, $l=\overline{-N_2,N_2}$. We introduce the Cartesian coordinate system with the origin inside the central element S_{00} of the grating (see Fig. 1) and direct the axis z orthogonally to the plane of the grating. Let (x_{jl},y_{jl},z_{jl}) be the coordinate system connected with the grating element with the numbers j and l. We let $(x^0_{jl},y^0_{jl},0)$ denote the coordinates of the origin of the coordinate system connected with the corresponding element of the grating in the general system of coordinates.

Let the boundary conditions be posed on the surfaces S_{jl}:

$$\left(\vec{n}_{jl}\times\vec{E}\right)\Big|_{S_{jl}}=0,\quad j=\overline{-N_1,N_1},\quad l=\overline{-N_2,N_2}, \tag{1}$$

where $\vec{n}_{jl}$ is the unit vector of outer normal on the surface S_{jl}. The secondary field outside the grating obeys the homogeneous Maxwell equations

$$\nabla\times\vec{E}^1=-ik\zeta\vec{H}^1,\nabla\times\vec{H}^1=\frac{ik}{\zeta}\vec{E}^1, \tag{2}$$

where $k=\omega\sqrt{\varepsilon\mu}$ is the wave number and $\zeta=\sqrt{\mu/\varepsilon}$ is the wave impedance of the medium. The Sommerfeld condition is satisfied at infinity:

$$\left(\vec{E}^1\times\frac{\vec{r}}{r}\right)+\zeta\vec{H}^1=o\left(\frac{1}{r}\right),\left(\vec{H}^1\times\frac{\vec{r}}{r}\right)-\frac{1}{\zeta}\vec{E}^1=o\left(\frac{1}{r}\right)\langle r\to\infty.\rangle \tag{3}$$

First, we consider the approach based on PEM. We present the secondary field outside the grating as the sum

$$\vec{E}^1=\sum_{j=-N_1}^{N_1}\sum_{l=-N_2}^{N_2}\vec{E}^1_{jl},\quad \vec{H}^1=\sum_{j=-N_1}^{N_1}\sum_{l=-N_2}^{N_2}\vec{H}^1_{jl}, \tag{4}$$

where $\vec{E}^1_{jl}$, $\vec{H}^1_{jl}$ denote the field scattered at the element with the numbers j and l. As is known, the following asymptotic relations in the far zone ($kr_{jl}>>1$) are valid [8]:

$$\vec{E}^1_{jl}=\frac{\exp(-ikr_{jl})}{r_{jl}}\vec{F}^E_{jl}(\theta_{jl},\varphi_{jl})$$

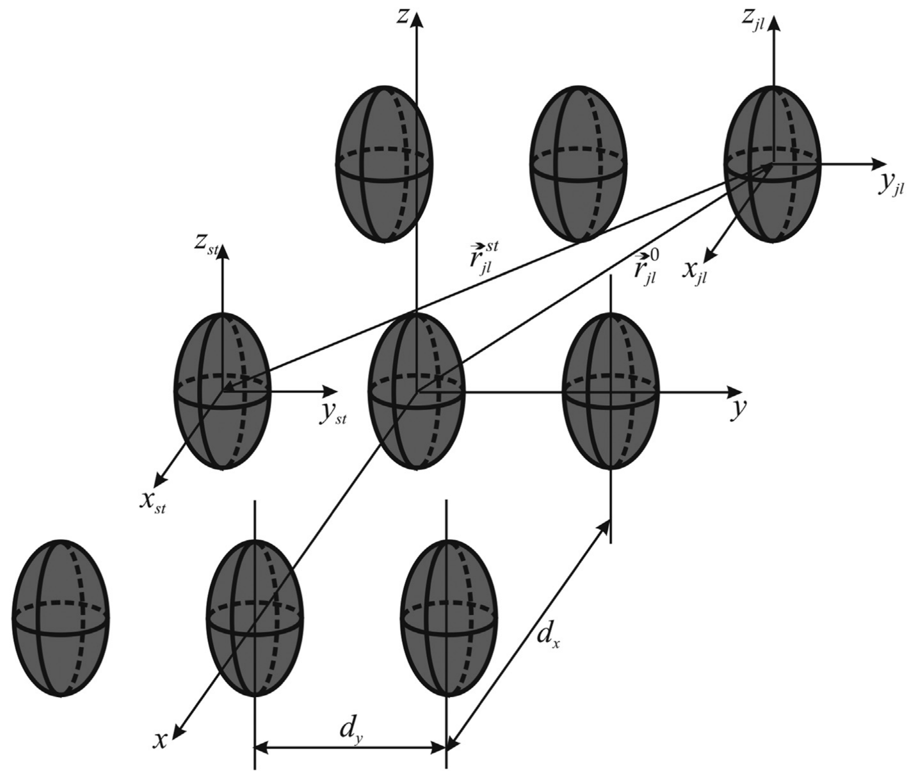

Fig. 1. Geometry of the problem.