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Procedures for the measurement of the extinction cross section of one particle using a Gaussian beam

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ABSTRACT

Two procedures for the measurement of the extinction cross section (ECS) of one particle using a slightly focused Gaussian beam have been introduced and numerically tested. While the first one relies on previously introduced ideas and has close connection with the optical theorem, the second procedure is new and is mostly related with light measurements where the detector collects much of the energy of the incident beam.

Both procedures prove to be valid and somehow complementary up to particle sizes of the order of the beam waist, thus enlarging the capability of simple measurement set-ups based on Gaussian beams for the estimation of the ECS of one particle.

1. Introduction

The calculation and measurement of the ECS of one singe particle is a common problem in light scattering theory [1,2]. The generalization of the definition of ECS to the case of non-plane incident wave is also a topic widely addressed. Of course, one of the cases of major interest is for Gaussian beams. Without going very back in time, some relevant references are (in chronological order) [3–9]. A complete review of the topic is done in [10].

Nowadays, the ease of use of Gaussian beams focused on one particle suggests that it would be useful to devise approximate experimental procedures for the measurement of the ECS of the particle based on that configuration. In our work we will consider a linearly polarized Gaussian beam slightly focused (divergence angles up to a maximum of 10°) as the incident excitation.

In this context, we propose two approximate methods for the measurement of the ECS of a single particle by means of the detection of light without and with the particle placed on the focus of the incoming Gaussian beam. One of the procedures was already introduced in Ref. [5] and relies on the light detection only in a small angle in the forward direction. Conversely, the second procedure that we will propose will be more adequate when light collecting angles are wider.

In Ref. [5], the validity of several approximate expressions for the calculation of the ECS by using Gaussian beams is discussed in great detail, both from an analytical and from a numerical point of view. Restricting ourselves to numerical tests, the present work checks the accuracy of the two methods we will propose by means of computer calculations and analyzes how they compare with analytical results for spherical particles using Mie theory. The work is based on the explicit numerical calculation of the Poynting vector of the waves reaching the light detector. This detector is considered to be of finite aperture, subtending a well-known angle from the center of the Gaussian beam, just the precise position where the particle is placed. For this purpose, specific and precise numerical methods have been developed. The necessary procedures

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for the detailed development of our ideas are presented as follows:

2. Development of specific numerical methods to handle the problem

For the calculation of the Poynting vector at any point in space, the interference between the beam illuminating the particle and the subsequently scattered field must be explicitly formulated.

In our thought experiment (Gedankenexperiment) the illumination is performed with a Gaussian beam with small divergence angle $\sin(\delta_0) = \mathrm{NA}_{\mathrm{eff}}$, whose mathematical expression (within the standard paraxial theory) is:

$$E(z,\rho) = E_0 \frac{\omega_0}{\omega(z)} \exp\left(\frac{-\rho^2}{\omega^2(z)}\right) \exp i\left(\frac{k\rho^2}{2R(z)} - \Phi_G(z)\right) \exp i(kz), \tag{1}$$

where ω_0 is the beam radius in the focal plane, $R(z) = z\left(1+\left(\frac{z_R}{z}\right)^2\right)$, $z_R = \frac{k\omega_0^2}{2}$, $\Phi_G(z) = \tan^{-1}\left(\frac{z}{z_R}\right)$, ρ is the radial distance from the axis and k the wave-vector.

The previous formulae assume the Z axis to be beam axis and the X and Y directions to be in the transverse directions. We will assume that the Gaussian beam is linearly polarized, with the electric field having always the direction of the constant unit vector $\hat{e}_x$. Within this approximation, the corresponding magnetic field is:

$$H = \frac{\hat{e}_z \times E}{c\mu_0}, \tag{2}$$

with c the speed of light and μ_0 the magnetic permeability of vacuum. Given the wavelength λ, power P_0 and $\mathrm{NA}_{\mathrm{eff}} = \sin(\delta_0)$ the peak amplitude E_0 can easily be found by using

$$\omega_0 = \frac{\lambda}{\pi\delta_0} \tag{3}$$

and

$$P_0 = \frac{\pi E_0^2 \omega_0^2}{4c\mu_0} \tag{4}$$

We plan to use the Mie theory for the calculation of the scattered field, but this theory is developed for a spherical particles excited by a linearly polarized plane wave (of amplitude E_p). Generalized Mie theories [10] can tackle the situation of spatially inhomogeneous illumination. Yet, if the particle can be considered as homogeneously polarized, the use of Mie theory is well justified [2]. In any case, since our illumination is not a plane wave, we have to estimate E_p for the particle, when it is being excited by the Gaussian beam.

When the particle (with radius a) is centered in the beam waist, even when the particle is small, taking $E_p = E_0$ is not the best choice since, as the Gaussian profile has a maximum on axis, this assumption always overestimates the value for E_p. We propose calculate E_p as follows.

a. Find the power incident on a centered circle of radius a,

$$P_a = P_0\left(1 - \exp\left(\frac{-2a^2}{\omega_0^2}\right)\right) \tag{5}$$

b. Find the constant ('mean') value for the electric field E_m that corresponds to the flux of the power P_a across the area πa^2; this is

$$E_m = \sqrt{\frac{2P_a\eta_0}{\pi a^2}}, \text{ with } \eta_0 = 377\ \Omega. \tag{6}$$

c. Assume $E_p = E_m$.

Besides, this approach does not require the size of the particle to be very small. Once E_p is estimated, the Mie theory can be used, giving for the scattered field

$$E_s(r,\theta,\phi) \approx E_p \frac{\exp\ i(kr)}{-ikr} X(\theta,\phi). \tag{7}$$

The Mie theory provides the calculation of the angular term, the vector scattering amplitude $X(\theta,\phi)$, by assuming a spherical shape for the particle, with radius a. This topic is very well known; in Ref. [2] all important details can be found. Particularly, in our work we will make continuous use of the result regarding to the number of terms (in the final series development) that is enough for an accurate representation of the scattered field [11]. This result relates explicitly the number of terms to the radius of the particle (more terms required as the particle gets bigger). Thus, we consider our calculations of the scattered field as 'exact' since we have fulfilled the requirements imposed by the condition discussed in Ref. [11].

According to our previous choice for the axis, the scattered field will have the three spatial components but, clearly the longitudinal one (Z) will be negligible in the far field with respect to the other two. Thus, finally, when there is a particle present in the path of the Gaussian beam, we will consider the Jones vector of the total field reaching the light detector in the far zone to be

$$\begin{pmatrix} E_G + E_s/_x \\ E_s/_y \end{pmatrix}, \tag{8}$$

where E_s is the scattered field generated by the Gaussian beam E_G.

This formulation shows that, since the Cartesian components of the scattered field (in the far field) are being calculated, we can calculate the interferences (sum) between this scattered field and the incident Gaussian beam. The scenario is like in Figure 3.7 of Ref. [2], with the difference of assuming illumination by means of a Gaussian beam, not with a plane wave like there (see Fig. 1).

A careful analysis of Fig. 1 illustrates that the computation of the Poynting vector of the resulting field on the points of the sphere and subsequent integration over the area defined by the aperture of the detector, allows us to calculate the power collected by this detector. According to Fig. 1, the collecting area will be defined on the imaginary