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Multiple scattering by a collection of randomly located obstacles – numerical implementation of the coherent fields

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ABSTRACT

A numerical implementation of a method to analyze scattering by randomly located obstacles in a slab geometry is presented. In general, the obstacles can be of arbitrary shape, but, in this first implementation, the obstacles are dielectric spheres. The coherent part of the reflected and transmitted intensity at normal incidence is treated. Excellent agreement with numerical results found in the literature of the effective wave number is obtained. Moreover, comparisons with the results of the Bouguer–Beer (B–B) law are made. The present theory also gives a small reflected coherent field, which is not predicted by the Bouguer–Beer law, and these results are discussed in some detail.

1. Introduction

Electromagnetic scattering by randomly located objects are frequently encountered in science. It is an important issue in terrestrial and atmospheric research, biomedical and life sciences, astrophysics, nanotechnology, just to mention a few. The literature is comprehensive, and we refer to the textbook literature and references therein, see e.g. [1–9] for a survey of the field.

The literature contains several methods of computing the effective wave number k_{eff} for a half space containing a collection of random spheres, see e.g. [10–17] and [3, Chapter 6], and references therein. The effective wave number is obtained by solving a determinant relation and there are in general many solutions. The new method presented in a recent paper, [18], does not suffer from this deficiency and it is able to compute the coherent transmitted and reflected fields from a finite slab containing random scatterers. In this paper, results are presented for slabs with different thicknesses and spherical scatterers with the relative permittivity $\epsilon_r = 1.33^2$ which corresponds to fresh water at optical frequencies (refraction index $m = 1.33$). Both the electrical size of the spheres and the volume fraction are varied.

2. Theory

The theory of electromagnetic scattering by an ensemble of finite scatterers is comprehensive, and excellent reviews of the topic are found in [1–7,19]. The underlying theoretical treatment of the problem handled in this paper is presented in detail in Kristensson [18]. The purpose of this section is to review and highlight some of the more important steps in the theory. For a more complete reference we refer to Kristensson [18].

We simplify the theoretical results in [18] to a geometry of a slab, $z \in [0, d]$, and to spherical scatterers of radius a (dielectric or perfectly conducting). These assumptions simplify the results considerably, and make the numerical implementation less demanding. The geometry is depicted in Fig. 1. Notice that the domain of possible locations of local origins, $[z_0, z_d]$, which defines the domain V_s, is

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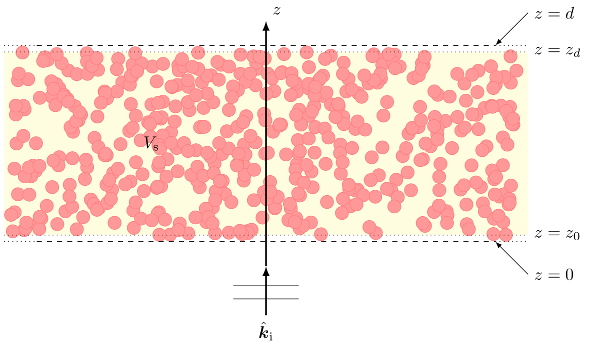

Fig. 1. The geometry of the stratified scattering region. The yellow region denotes the region V_s, which is the domain of possible locations of local origins, i.e., the interval $[z_0, z_d]$. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

slightly smaller than the extent of the slab, i.e., the interval $[z_0, z_d] = [a, d-a]$. Vectors are denoted in italic boldface and matrices in roman boldface. A caret over a vector denotes a vector of unit length. In this paper, we also adopt the multi-index notation $n = \tau\sigma ml$, where the integer indices $\tau = 1, 2$, $\sigma = \mathrm{e}, \mathrm{o}$ (even and odd in the azimuthal angle), $m = 0, 1, \ldots, l$ and $l = 1, 2, 3, \ldots$.

Assume the incident field on the slab is

$$\boldsymbol{E}_{\mathrm{i}}(z) = \boldsymbol{E}_0 \mathrm{e}^{\mathrm{i}k_0 z}$$

The coherent part of the total electric field on either side of the slab is

$$\langle \boldsymbol{E} \rangle(z) = \begin{cases} \boldsymbol{E}_{\mathrm{t}} \mathrm{e}^{\mathrm{i}k_0 z}, & z > d \\ \boldsymbol{E}_0 \mathrm{e}^{\mathrm{i}k_0 z} + \boldsymbol{E}_{\mathrm{r}} \mathrm{e}^{-\mathrm{i}k_0 z}, & z < 0 \end{cases}$$

where the reflected and transmitted amplitudes, $\boldsymbol{E}_{\mathrm{t}}$ and $\boldsymbol{E}_{\mathrm{r}}$, respectively, are given as

$$\boldsymbol{E}_{\mathrm{t}} = \boldsymbol{E}_0 + \frac{2\pi n_0}{k_0^2} \sum_{l=1}^{\infty} \mathrm{i}^{-l} \sqrt{\frac{2l+1}{8\pi}} \Big(\hat{\boldsymbol{x}} \int_{z_0}^{z_d} \mathrm{e}^{-\mathrm{i}k_0 z'} \left(\langle f_{1\mathrm{o}1l} \rangle (z') + \mathrm{i} \langle f_{2\mathrm{e}1l} \rangle (z') \right) \mathrm{d}z' - \hat{\boldsymbol{y}} \int_{z_0}^{z_d} \mathrm{e}^{-\mathrm{i}k_0 z'} \left(\langle f_{1\mathrm{e}1l} \rangle (z') - \mathrm{i} \langle f_{2\mathrm{o}1l} \rangle (z') \right) \mathrm{d}z' \Big) \tag{1}$$

and

$$\boldsymbol{E}_{\mathrm{r}} = \frac{2\pi n_0}{k_0^2} \sum_{l=1}^{\infty} \mathrm{i}^{l} \sqrt{\frac{2l+1}{8\pi}} \Big(\hat{\boldsymbol{x}} \int_{z_0}^{z_d} \mathrm{e}^{\mathrm{i}k_0 z'} \left(\langle f_{1\mathrm{o}1l} \rangle (z') - \mathrm{i} \langle f_{2\mathrm{e}1l} \rangle (z') \right) \mathrm{d}z' - \hat{\boldsymbol{y}} \int_{z_0}^{z_d} \mathrm{e}^{\mathrm{i}k_0 z'} \left(\langle f_{1\mathrm{e}1l} \rangle (z') + \mathrm{i} \langle f_{2\mathrm{o}1l} \rangle (z') \right) \mathrm{d}z' \Big) \tag{2}$$

in terms of the number density n_0 and the (unknown) coefficients $\langle f_n \rangle(z)$. The coefficients $\langle f_n \rangle(z)$ are the solution to a system of linear Fredholm integral equations of the second kind in z [20], *viz.*,

$$\langle f_n \rangle(z) = \mathrm{e}^{\mathrm{i}k_0 z} \sum_{n'} T_{nn'} a_{n'} + k_0 \int_{z_0}^{z_d} \sum_{n'} K_{nn'}(z - z') \langle f_{n'} \rangle (z') \mathrm{d}z', \quad z \in [z_0, z_d] \tag{3}$$

where the transition matrix of the scatterers is denoted $T_{nn'}$ and where the kernel $K_{nn'}(z)$ can be expressed in terms of spherical waves [18,21]. The explicit form of the kernel $K_{nn'}$ is ($\boldsymbol{\rho} = x\hat{\boldsymbol{x}} + y\hat{\boldsymbol{y}}$)

$$K_{nn'}(z) = \frac{n_0}{k_0} \sum_{n''} T_{nn''} \iint_{\mathbb{R}^2} g(|\boldsymbol{\rho} - z\hat{\boldsymbol{z}}|) \mathcal{P}_{n''n'}(k_0(\boldsymbol{\rho} - z\hat{\boldsymbol{z}})) \mathrm{d}x\mathrm{d}y, \quad |z| < z_d - z_0$$

where $g(r)$ is the pair distribution function [2,22–24] and $\mathcal{P}_{nn'}(k_0\boldsymbol{d})$ is the translation matrix for the outgoing spherical vector waves [25,26]. The most simple pair distribution function is the hole correction (HC), $g(r) = H(r - 2a)$, where $H(x)$ is the Heaviside function and a is the radius of the spheres. The double integral in the definition of the kernel can be solved analytically for the hole correction in terms of a series of spherical waves [21]. More complex distributions functions, e.g. the hypernetted-chain equation, the Percus–Yevick approximation (P-YA), the self-consistent approximation, and the Monte Carlo calculations, are not employed in this paper [2,22–24].

The spherical scatterers are completely characterized by the transition matrix $T_{nn'}$, which for a spherical scatterer is diagonal in all its indices. The coefficients a_n are the expansion coefficients of the incident plane wave in spherical vector waves [18,26]. If the incident direction is along the positive z-direction, i.e., $\hat{\boldsymbol{k}}_i = \hat{\boldsymbol{z}}$, these are ($\sigma = \mathrm{e}$ is the upper line and $\sigma = \mathrm{o}$ is the lower line)

$$\begin{cases} a_{1\sigma ml} = -\mathrm{i}^{l} \delta_{m1} \sqrt{2\pi(2l+1)} \left(\hat{\boldsymbol{z}} \times \begin{Bmatrix} \hat{\boldsymbol{x}} \\ \hat{\boldsymbol{y}} \end{Bmatrix} \right) \cdot \boldsymbol{E}_0 \\ a_{2\sigma ml} = -\mathrm{i}^{l+1} \delta_{m1} \sqrt{2\pi(2l+1)} \begin{Bmatrix} \hat{\boldsymbol{x}} \\ \hat{\boldsymbol{y}} \end{Bmatrix} \cdot \boldsymbol{E}_0 \end{cases} \quad \hat{\boldsymbol{k}}_i = \hat{\boldsymbol{z}}$$

where the vector $\boldsymbol{E}_0$ denotes the polarization state in the x–y plane.

The complex valued transmission and reflection coefficients that maps the incident field to the transmitted and reflected fields are

$$\boldsymbol{E}_{\mathrm{t}} = t\boldsymbol{E}_0, \quad \boldsymbol{E}_{\mathrm{r}} = r\boldsymbol{E}_0 \tag{4}$$

respectively. The transmissivity T and the reflectivity R of the slab are given by

$$T = \frac{|\boldsymbol{E}_{\mathrm{t}}|^2}{|\boldsymbol{E}_0|^2}, \quad R = \frac{|\boldsymbol{E}_{\mathrm{r}}|^2}{|\boldsymbol{E}_0|^2} \tag{5}$$

3. Numerical implementation

To compute the reflection and the transmission coefficients of the slab, we need to solve (3) for given geometrical and material data. The unknown quantity, $\langle f_n \rangle(z)$, is evaluated at equally spaced points, $z = z_1, z_2, \ldots, z_p$, in the interval $[z_0, z_d]$, and the integral in (3) is evaluated by the use of Simpson's quadrature at the points of discretization. The spatially discretized vector $\langle f_n \rangle$ is denoted $\boldsymbol{F}$. Remembering that n is a multi-index of $n = \tau\sigma ml$, the