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Application of alternative synthetic kernel approximation to radiative transfer in regular and irregular two-dimensional media

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Alternative synthetic kernel (ASK_N) approximation, just as the standard SK_N method, is derived from the radiative integral transfer equations in full 3D generality. The direct and diffuse terms of thermal radiation appear explicitly in the radiative integral transfer equations as surface and volume integrals, respectively. In standard SK_N method, the approximation is employed to the diffuse terms while direct terms are evaluated analytically. The alternative formulation differs from the standard one in that the direct radiation wall contributions are also approximated with the same spirit of the synthetic kernel approximation. This alternative formulation also yields a set of coupled partial differential-the ASK_N–equations which could be solved using finite volume methods. This approximation is applied to radiative transfer calculations in regular and irregular two-dimensional absorbing, emitting and isotropically scattering media. Four benchmark problems-one rectangular and three irregular media-are considered, and the net radiative flux and/or incident energy solutions along the boundaries are compared with available exact, standard discrete ordinates S_4 and S_{12}, modified discrete ordinates S_4, Monte Carlo and collocation spectral method to assess the accuracy of the method. The ASK_N approximation yields ray effect free incident energy and radiative flux distributions, and low order ASK_N solutions are generally better than those of the high order standard discrete ordinates method.

1. Introduction

Radiative transfer of energy plays an important significant role in thermal design of many industrial systems such as combustion chambers, furnaces, ovens, drying processes, boilers, gas turbines, etc. The computation of radiative heat transfer in participating media requires the solution of the radiative transfer equation (RTE) which is cast as an integro-differential equation for the radiation intensity. The RTE depends on space (x, y and z) and angular (θ and φ) variables. Herewith the inherent nature of the RTE, it is generally difficult to obtain analytical solutions for multi-dimensional medium. Consequently, solution of the RTE requires approximate methods.

Exact solutions of RTE are obtained from the solution of the radiative integral transfer equations (RITEs)–an alternative form of the RTE. The RITEs are obtained by integrating the RTE over all solid angle to eliminate the angular variables [1–5]. This procedure leads to a set of coupled integral equations for the incident energy and radiative heat flux. Analytical solutions of the RITE is

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Nomenclature

D_n	artificial diffusion coefficient defined as μ_n^2/β (m)
$E_k(x)$	kth order exponential integral function
G	incident energy (W m^{-2})
G_n	scalar function defined by Eq. (10).
G_n^*	scalar function defined by Eq. (16).
f_G, $\mathbf{f_q}$	direct radiant energy contributions defined by Eqs. (4) and (5).
I_w, I_0	incident wall radiation (W m^{-2} sr^{-1})
I_b	blackbody radiation intensity (W m^{-2} sr^{-1})
$K_n(x)$	nth order modified Bessel function
K_n	approximated kernels (dimensionless)
mfp	mean free path
$\mathbf{q}_n$	vector function defined by Eq. (11).
$\mathbf{q}_n^*$	vector function defined by Eq. (16).
R	half thickness of plane and/or radius of cylinder/sphere (m)
$\mathbf{r}$	position vector inside a medium (m)
$\mathbf{r}_w$	position vector on a surface (m)
S_0	isotropic medium source defined by Eq. (3).
S, $d\mathbf{S}$	surface and normal vector of enclosing wall
T	temperature (K)
$\mathbf{q}$	net radiative heat flux (W m^{-2})
w_n	weights of half- or full-range Gauss-Legendre quadratures
x,y,z	coordinates variables (m)

Greek symbols

β	extinction coefficient (m^{-1})
δ	Delta Dirac function
τ	optical variables (dimensionless)
κ	absorption coefficient (m^{-1})
ρ	distance between two points (m)
μ_n	abscissas of half- or full-range Gauss-Legendre quadratures
ω_0	scattering albedo (dimensionless)
σ_s	scattering coefficient (m^{-1})

available only for some simple and idealized cases, while exact solutions of higher dimension and complexity could be obtained from the grid-independent numerical solutions. Although the RITE formalism is dimensionally easier to handle, resulting integral kernels are singular in nature which should be derived and/or treated analytically for each unique geometry [6–17]. Furthermore, numerical treatments of the RITEs yields system of linear equations with dense matrices which, in turn, impose restrictions on the computer memory and cpu-time even for simplest 2D/3D enclosures. These undesired computational features of the RITEs make the integral transfer approaches unsuitable to be employed to practical engineering analysis. Therefore, solutions of the RITEs are generally sought for benchmarking purposes.

The spherical harmonics (or P_N) approximation is one the earliest methods applied to the solution of integro-differential equations in both neutron transport theory and thermal radiative transfer. It is based on separating the spatial and angular dependence of the radiative intensity by expressing the solution in terms of a set of angular basis functions [18]. This representation leads to an infinite set of coupled first-order differential equations. In practice, the approximate solutions are constructed by truncating the expansion to a finite subset of spherical harmonics. When the order of approximation is increased, the solutions converge slowly while the mathematical complexities increase dramatically [19–28]. The primary disadvantages of P_N rest in the degree of approximation needed for optically thin media with large scattering; however, P_1 (or *diffusion*) approximation is simple and sufficient for the analysis of optically thick media. For this reason, it has been incorporated into most commercial software as a standard feature.

Another approximation which uses the Legendre polynomials-so-called double spherical harmonics (or P_{NN}, double P_N or DP_N) approximation-was proposed by Yvon [29] to solve neutron transport problems. Same approach was proposed by Schuster [30] and Schwarzschild [31] (*Schuster-Schwarzschild* or *two-flux approximation*) to solve radiative transfer problems. In this method, instead of one expansion for all angles, separate expansions for the radiation traveling in the forward and backward directions are used which allows a discontinuity in angular distribution to be treated more accurately. Even with the lowest order of approximations, the method generally provides a good fit to a rapidly varying angular quantities. In radiative transfer analyses, the DP_N approximation has found application in 1D planar and 1D spherical media [32–35]. Due to the complexities of the resulting differential equations in multi-dimensional geometries, the DP_N method, like P_N, has not become a popular method.

The discrete ordinates method (DOM) was introduced by Chandrasekhar [36] to compute radiation transfer in stellar atmospheres. The present DOM evolved from the angular discretization of Lee [37] and Lathrop [38,39] to solve the neutron transport equation. Fiveland [40–42] and Truelove [43,44] introduced the method to the radiative transfer computations, and it quickly became a popular method of computing radiative transfer in multi-dimensional enclosures [45–47]. In the DOM, the space is divided into spatial grids while the direction is handled by computing the quantities in discrete directions introduced through a numerical quadrature to the integral terms over the solid angle. The DOM equations are simple to derive and lead to efficient computational algorithms. The computation time depends on the geometric complexity, grid resolution and the number of discrete directions. The DOM with space discretized using finite volume method (FVM) have received plenty of attention and are today probably the most popular RTE solvers together with the diffusion (P_1) approximation. Some version of FVM DOM have been