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General description of circularly symmetric Bessel beams of arbitrary order

Jia Jie Wang [a,b,*], Thomas Wriedt [b], James A. Lock [c], Lutz Mädler [b]

[a] *School of Physics and Optoelectronic Engineering, Xidian University, 710071 Xi'an, China*
[b] *Foundation Institute of Material Science (IWT), Department of Production Engineering, University of Bremen, Badgasteiner Str. 3, 28359 Bremen, Germany*
[c] *Department of Physics, Cleveland State University, Cleveland, OH 44115, USA*

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ABSTRACT

A general description of circularly symmetric Bessel beams of arbitrary order is derived in this paper. This is achieved by analyzing the relationship between different descriptions of polarized Bessel beams obtained using different approaches. It is shown that a class of circularly symmetric Davis Bessel beams derived using the Hertz vector potentials possesses the same general functional dependence as the aplanatic Bessel beams generated using the angular spectrum representation (ASR). This result bridges the gap between different descriptions of Bessel beams and leads to a general description of circularly symmetric Bessel beams, such that the Davis Bessel beams and the aplanatic Bessel beams are merely the two simplest cases of an infinite number of possible circularly symmetric Bessel beams. Additionally, magnitude profiles of the electric and magnetic fields, the energy density and the Poynting vector are displayed for Bessel beams in both paraxial and nonparaxial cases. The results presented in this paper provide a fresh perspective on the description of Bessel beams and cast some insights into the light scattering and light-matter interactions problems in practice.

1. Introduction

Along with a wide application of various laser-based optical instruments, such as Phase Doppler Anemometry (PDA), Laser Doppler Velocimetry (LDV), Optical Tweezers and many others, the investigation of interactions between shaped laser beams and small particles becomes a very hot topic in recent years, which attracts attention of researchers from lots of areas [1,2]. In the analysis of various shaped beams, there has been an increasing interest in Bessel beams which were introduced by Durnin and co-workers [3,4] almost three decades ago. Although ideal Bessel beams cannot be generated in reality, high quality quasi-Bessel beams can be generated using an axicon lens [5,6], spatial light modulator (SLM) [7,8], or a combination of an axicon and a spatial light modulator [9]. The geometry of a quasi-Bessel beam generated by an axicon is shown in Fig. 1. Due to the special properties of Bessel beams, including propagation invariance [10], self-reconstruction, long focal depth of field [11,12] as well as the transfer of orbital angular momentum and spin angular momentum to matter [13], prospective applications of Bessel beams can be found in various fields, such as optical communication, accurate optical measurement, optical manipulation of small particles, and imaging [14,15].

The description of shaped beams is a fundamental issue. It plays a key role in the analysis of beam properties,

* Corresponding author at: School of Physics and Optoelectronic Engineering, Xidian University, 710071 Xi'an, China.
E-mail address: wangjiajie@xidian.edu.cn (J.J. Wang).

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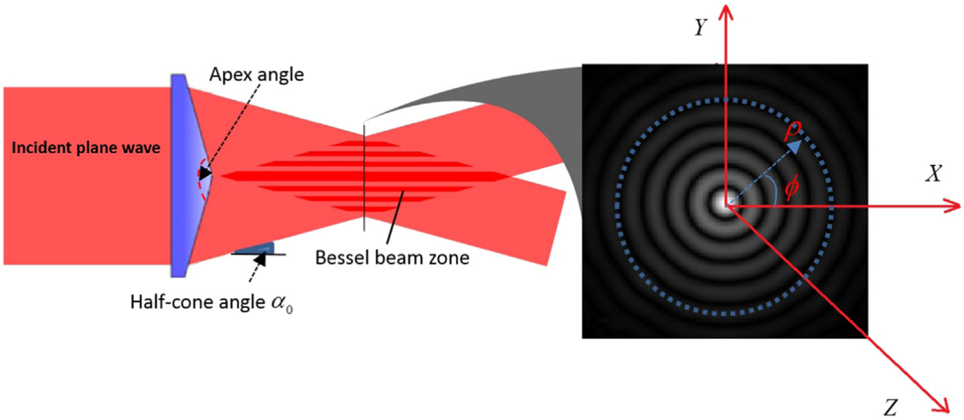

Fig. 1. Geometry of a quasi-Bessel beam generated using an axicon lens. Half-cone angle of the Bessel beam is α_0. A Cartesian coordinate system $(X,\ Y,\ Z)$ and a corresponding cylindrical coordinate system $(\rho,\ \phi,\ z)$ are used.

beam propagation as well as light-matter interactions. As an exact solution of the scalar wave equation, a basic description of Bessel beams in scalar version [3,4] was applied when the Bessel beam was introduced. For an on-axis Bessel beam propagating along the z axis, the general expression for the scalar field is described by $\psi(\rho,\phi,z;t)=\psi_0 J_n(k_t\rho)e^{in\phi}e^{-i(k_z z-\omega t)}$, where ψ_0 is the amplitude of the field, and $J_n(\cdot)$ is the n-order Bessel function of the first kind. The parameters $\rho=\sqrt{x^2+y^2}$ and $\phi=\tan^{-1}(y/x)$ are the radial distance and the azimuthal angle in the transverse plane (x,y), respectively. The transverse and longitudinal wave numbers are $k_t=k\ \sin\ \alpha_0$ and $k_z=k\ \cos\ \alpha_0$, respectively. The wavenumber is k, and α_0 is the half-cone angle of the Bessel beam which is defined with respect to the axis of wave propagation. If $\alpha_0=0$, the scalar Bessel beam reduces to a scalar plane wave. The time-dependent part of the wave $\exp(i\omega t)$ is used and omitted throughout in this paper, with ω being the angular frequency. So far, there are a number of studies based on the scalar field description [16–19], which gives satisfactory results under the paraxial conditions, e.g the spot size of the beam is much larger than the wavelength. A vectorial treatment is required for an adequate description of polarized electromagnetic wave radiation and scattering, especially in nonparaxial cases where tightly focused Bessel beams are used, e.g. in optical tweezers where small particles are manipulated by a tightly focused laser beam [11,13,20]. The intensity profile of a scalar Bessel beam is circularly symmetric, while the intensity profile of a polarized Bessel beam can be circularly symmetric or asymmetric [21,22]. The Bessel beams with a circularly symmetric distribution of energy density have been called circularly symmetric Bessel beams, whose Poynting vector component along the propagation direction is also circular symmetric.

Although ideal Bessel beams can hardly be generated in reality, it is common practice to start with the simplest theoretical assumption of idealized fields, which can cast insights into practical analysis where quasi-Bessel beams are applied. Several vectorial approaches have been proposed to describe ideal Bessel beams, with exact vectorial solutions to the Maxwell's equations. Bouchal and Olivík [23] derived expressions for polarized Bessel beams of arbitrary order as the solution to the vector Helmholtz equation, in which radial, azimuthal, circular and linear polarizations were analyzed. Recently, facilitated by the application of the Hertz vector potential [24], the Bessel beams of transverse magnetic (TM) and transverse electric (TE) mode [25,26] and the linearly and circularly polarized Bessel beams [27] were derived in a rather simple way. This is due to the fact that the potentials are more fundamental quantities than the electric and magnetic fields. Once the potentials are known, the fields can be obtained by differentiation. In this procedure the derivation of the fields is implemented in the Lorenz condition when linearly polarized vector potentials are used, which is similar to the procedure used by Davis [28] for the development of a Gaussian beam model. Thus the Bessel beam derived using the vector potential has been called a Davis Bessel beam [29] to distinguish it from the Bessel beam obtained using the angular spectrum representation (ASR), which is commonly called an aplanatic Bessel beam since it was originally proposed for an aplanatic optical system [30]. The ASR method was introduced by Cizmar et al. [31] to describe a focused zero-order aplanatic Bessel beam generated by an axicon lens. Later it was extended to the description of Bessel beams of higher-order by Chen et al. [32,33]. More detailed expressions of higher-order aplanatic Bessel beams were presented by Mitri et al. [34] in a study of resonance scattering of a dielectric sphere and used recently by Yang and Li [35] to calculate the optical force exerted on a Rayleigh particle.

Although various descriptions of polarized Bessel beams derived using different approaches are available in the literature, different approaches give seemingly different answers for the fields. This situation casts confusion and sometimes leads to a misuse of Bessel beam expressions. Thus a clear picture of the connection between different descriptions of Bessel beams is necessary for easier applications in practice as well as providing some insights into the nature of an ideal Bessel beam. This was recently done for a zero-order Bessel beam by Lock [29]. The aplanatic Bessel beam of zero-order generated with the ASR was found to have a same functional dependence