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Evaluation of three different radiative transfer equation solvers for combined conduction and radiation heat transfer

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ABSTRACT

This work investigates the performance of P1 method, FVM and SP3 method for 2D combined conduction and radiation heat transfer problem. Results based on the Monte Carlo method coupled with the energy equation are used as the benchmark solutions. Effects of the conduction-radiation parameter and optical thickness are considered. Performance analyses in term of the accuracy of heat flux and temperature predictions and of computing time are presented and analyzed.

1. Introduction

Radiation heat transfer is important in many industrial devices, especially those with high temperature. It is important to both the evolution of the temperature field (through the volumetric radiative heat source) and the wall total heat flux. An overall thermal analysis often needs to take conduction, convection and radiation into consideration. Unlike conduction and convection, the radiative transfer equation (RTE) is an integro-differential equation, which depends on two angular dimensions, and this makes it difficult to solve. Even in the absence of scattering, combined heat transfer modes with radiation are sometimes difficult to converge due to the high non-linearity that radiation brings. Given this condition, only numerical methods are applicable for real problems.

The purpose to this work is to provide insight into the factors that are necessary to choose an appropriate RTE solver for a combined model problem. Three RTE solvers will be used in a simple conduction problem to point out the important things to consider.

Many methods have been developed to solve the RTE, such as the Monte Carlo Method (MCM) [1–3], the zonal method [4], the diffusion approximation [5], the spherical harmonics method (PN) [6,7], the discrete transfer method (DTM) [8,9], the discrete ordinates method (DOM, or SN) [10–13], the finite volume method (FVM) [14,15], the finite element method (FEM) [16,17], the meshless method [18], and so on. Among them, PN (e.g. P1), DOM and FVM have gained popularity and are implemented into general purpose CFD software, such as OpenFOAM and ANSYS Fluent, while they leave the choice of method up to the user. Tencer and Howell [19] performed a parametric study of the accuracy of M1 method, P1 method and SN method for several 2D benchmark problems. It is shown that SN method suffers from ray effects for Case 1 and provides very accurate results for Case 3 and the P1 method error is less than the M1 method while larger than the SN method for these two cases for parameters considered in that work. Numerical simulations of combined problems often start from zero or uniform internal temperature with different wall temperature (discontinuous or continuous). Thus different RTE solvers coupled with the energy equation iteration may lead to quite different convergence

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behavior, that is, combined problem convergence may rely largely on the performance of RTE solvers.

Yuen and Takara [20] used the generalized exponential integral function to solve two dimensional combined conductive–radiative heat transfer. They concluded that the additive solution is an acceptable approach in estimating heat transfer and the diffusion approximation yields significant errors in both the temperature and heat flux predictions. Kim and Baek [21] studied the same problem using the central difference scheme for the heat diffusion equation and the discrete ordinates method for the RTE. They also included anisotropic scattering. Wu and Ou [22] took scattering into consideration for transient two-dimensional combined problems and investigated the influence of aspect ratio, scattering albedo and conduction-radiation parameters and concluded that the modified differential approximation is superior to the P1 method. The largest optical thickness considered was 5. Mishra et al. [23] investigated the computational efficiency of the collapsed dimension method and discrete transfer method for several radiation and combined mode heat transfer examples. These two methods were found to give the same results while the former was faster. Mishra [24] compared DTM, DOM and FVM methods for transient combined conduction and radiation heat transfer and found that DTM is the most time consuming while DOM is most efficient. Only an optical thickness of unity was considered. Wang et al. [25] developed a numerical iteration technique based on the flux conservation equation to solve 1D coupled conduction-radiation problem and showed that it converges as rapidly as Thomas's method. Naraghi and Saltiel [26] compared the performance of the conventional successive substitution method, accelerated fixed points, and Newton–Raphson methods, and they considered the effect of variable conductivity, while the optical thickness was unity. Recently, there are papers dealing with combinations of different energy solvers and RTE solvers for these combined problems, such as the spectral collocation method [27,28], natural element method [29,30], meshless method [31,32], lattice Boltzmann method (LBM) coupled with DOM and FVM [33–36], and LBM for both the energy equation and RTE [37]. These works mainly focus on developing new solvers for combined problems and how to choose an appropriate method for combined conduction and radiation heat transfer remains to be investigated.

Although comparisons of different RTE solvers exist for a wide range of parameters [19], Tencer [38] analyzed the advantages and disadvantages of implementing different RTE solvers into conjugate heat transfer codes, while there is little work reporting their performance for combined heat transfer [38,39]. Dombrovsky [40,41] analyzed the choice of method for RTE when considering inhomogeneous properties and scattering. The choice of method may depend on accuracy and computing time, which are functions of many parameters, e.g., optical thickness, conduction-radiation number, scattering albedo, spectral properties. This kind of problem has not been widely investigated and this motivates us to do this work.

This work aims to investigate the performance of P1 method, FVM and SP3 method for 2D combined conduction and radiation heat transfer. The P1 method and FVM are widely used for practical applications. The SP3 method is chosen in this work as it shows a distinct improvement over the P1 method with reasonable computing time [42]. Results obtained by MCM coupled with the energy equation serve as the benchmark solution, although MCM may introduce convergence instabilities considering its stochastic nature. It produced very smooth results for most of the cases studied. The results of accuracy in terms of heat flux, temperature, and computing time will be presented and analyzed in detail.

2. Mathematical formulations

2.1. Energy equation

For steady combined conduction and radiation heat transfer problems, the energy equation is

$$\nabla(k\nabla T) = \nabla\cdot\mathbf{q}_r \tag{1}$$

where k is the thermal conductivity, the left-hand side accounts for conduction, and the right-hand side accounts for radiation, which makes the combined heat transfer complicated. The local divergence of radiative heat flux can be calculated as:

$$\nabla\cdot\mathbf{q}_r = \kappa(4\sigma T^4 - G) \tag{2}$$

where κ is the absorption coefficient of the participating medium, σ is the Stefan-Boltzmann constant, and G is the incident radiation, which implicitly depends on the temperature field.

This divergence of radiative heat flux (or the radiative heat source), specifically, G is obtained by MCM, P1 method, FVM, and SP3 method in this work. Existence of fourth power in the radiation term introduces high non-linearity to the original simple diffusion equation. Incorporating the radiative heat source is usually done by loose coupling, in which $\nabla\cdot\mathbf{q}_r$ is computed based on initial conditions (basically temperature) by one of the RTE solvers. The radiative heat source is then inserted into the energy equation and a new temperature field is obtained, and the process is repeated until convergence. This separate solution procedure (or successive substitution) makes both the RTE and energy equation linear or slightly nonlinear and are commonly used in combined convection, combustion problems, in which the energy equation is more complicated.

2.2. P1 formulation

The P1 method is the simplest and most widely used RTE solver, and it consists of only one elliptic equation in terms of incident radiation, G

$$\frac{1}{3\kappa}\nabla\cdot\left(\frac{1}{\kappa}\nabla G\right) - G = -4\pi I_b \tag{3}$$

subject to the Marshak boundary condition:

$$-\frac{2-\varepsilon}{\varepsilon}\frac{2}{3\kappa}\mathbf{n}\cdot\nabla G + G = 4\pi I_{bw} \tag{4}$$

and the radiative heat flux is calculated by:

$$\mathbf{q}_r = -\frac{1}{3\kappa}\nabla G \tag{5}$$