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Notes

Truncation of the scattering phase matrix for vector radiative transfer simulation

Souichiro Hioki [a,*], Ping Yang [a], George W. Kattawar [a], Yongxiang Hu [b]

[a] Texas A&M University, College Station, TX, United States
[b] Climate Science Branch, NASA Langley Research Center, Hampton, VA, Unites States

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ABSTRACT

This short communication interprets the delta-fit technique in a context of similarity transformation and the correction to the source function, and derives the analogous form of the method to be applied for the scattering phase matrix. To adapt the delta-fit method to vector radiative transfer, the mathematically exact form of the similarity principle is used in the theoretical development. Some examples of relevant radiative transfer simulations are also presented for atmospheric ice particles. The performance of the adopted delta-fit method is comparable to the delta-M method with single scattering correction except for worse delta-fit performance for polarized radiance calculations in forward directions.

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1. Introduction

In the application of vector radiative transfer solvers to simulate, for example, ice cloud reflectivity and transmissivity, the presence of large particles triggers a trade-off between the accuracy and speed of the simulation. The scattering properties of such large particles cannot be appropriately represented for the radiance simulation with an affordable directional resolution (number of streams). Anisotropic scattering in the planetary atmosphere has been a major challenge, and Sobolev [1] discusses multiple methods of approximation for highly anisotropic scattering problems. Current common practice is to adapt a truncation technique to minimize error due to the reduced directional resolution used in the numerical radiative transfer simulations.

The δ-fit method developed by Hu et al. [2] fits the scattering phase function with a limited number of Legendre polynomials to accurately simulate the radiance. The method contrasts with the δ-M method developed by Wiscombe [3], which focuses on accurate flux simulation. Each method starts with the following plane-parallel radiative transfer equation:

$$-\mu\frac{\partial I}{\partial\tau}=-I(\tau,\mu,\phi)+\frac{\varpi(\tau)}{4\pi}\int_{-1}^{1}d\mu'\int_{0}^{2\pi}\mathcal{P}(\tau,\cos\Theta)I(\tau,\mu',\phi')d\phi'$$
$$+(1-\varpi)B(T), \quad (1)$$

where τ is optical thickness, μ is the cosine of the zenith angle, ϕ is the azimuth angle, $I(\tau,\mu,\phi)$ is the radiance propagating along the direction (μ,ϕ), ϖ is the single scattering albedo, $\mathcal{P}(\cos\Theta)$ is the phase function value at scattering angle Θ, and $B(T)$ is the blackbody emission at temperature T.

Each method uses the following similarity transformation, which defines scaled variables with an arbitrary

* Correspondence to: Department of Atmospheric Sciences, Texas A&M University, College Station, TX 77843, United States.
E-mail address: s.hioki@tamu.edu (S. Hioki).

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factor f in the form:

$$\tau^* = (1-f\varpi)\tau, \tag{2}$$

$$\varpi^* = \frac{1-f}{1-f\varpi}\varpi, \tag{3}$$

$$\mathcal{P}^*(\cos\Theta) = \frac{1}{1-f}\left[\mathcal{P}(\cos\Theta) - 4\pi f\delta(1-\cos\Theta)\right], \tag{4}$$

where $\delta(1-\cos\Theta)$ is the Dirac delta function peaked in the forward scattering direction. Substituting Eqs. (2)–(4) into Eq. (1) yields the same form for the radiative transfer equation but now for the scaled quantities indicated by the asterisk symbol; namely,

$$-\mu\frac{\partial I}{\partial\tau^*} = -I(\tau^*,\mu,\phi) + \frac{\varpi^*(\tau^*)}{4\pi}\int_{-1}^{1}d\mu'\int_0^{2\pi}\mathcal{P}^*(\tau^*,\ \cos\Theta)I(\tau^*,\mu',\phi')d\phi' + (1-\varpi^*)B(T). \tag{5}$$

In the classic approach (e.g. [4]), the original phase function $\mathcal{P}(\cos\Theta)$ is approximated by the sum of a delta function and a phase function without a forward peak $\mathcal{P}^*_{trunc}(\cos\Theta)$.

$$\mathcal{P}(\cos\Theta) \approx (1-f)\mathcal{P}^*_{trunc}(\cos\Theta) + 4\pi f\delta(1-\cos\Theta), \tag{6}$$

and the similarity transformation is derived as a consequence of the approximation. Eqs. (2)–(4) define an approximate radiative transfer problem in the classic approach.

However, the similarity transformation itself is mathematically exact [5,6], and it is beneficial for our discussion to accept the exact similarity transformation to unambiguously derive the vectorized form of the δ-fit method. Specifically, we interpret Eqs. (2)–(4) as definitions of scaled quantities τ^*, ϖ^*, and $\mathcal{P}^*(\cos(\Theta))$ without approximations. When Eqs. (2)–(4) are strictly satisfied, the solution of Eq. (5) gives an exact solution of the original radiative transfer equation Eq. (1).

By accepting the exact similarity transformation, truncation techniques are seen as optimization techniques. Truncation techniques simultaneously adjust parameter f and $\mathcal{P}^*_N(\cos(\Theta))$, which is a reconstructed phase function from N coefficients (c.f. Eq. (14)), to reduce the error in the flux or radiance. A similar interpretation is attempted by Mitrescu [6], who showed that the parameter f depends on the truncation method and the number of streams, in addition to the original phase function. Some systematic evaluations of these truncation techniques have been reported in the literature [7,8].

This short communication applies the framework discussed above, and interprets the δ-fit method as a correction to the source function, following the approach taken by Rozanov and Lyapustin [7]. The outcome of the theoretical development is an adaptation of the δ-fit method to vector radiative transfer. Some adjustments of the present method are provided for numerical implementation as a user-friendly computer program.

2. The δ-fit method

This section briefly summarizes the δ-fit method based on the original paper by Hu et al. [2]. The traditional δ-fit method starts with the expansion of the phase function in terms of Legendre polynomials $P_s(x)$,

$$\mathcal{P}(x) = \sum_{s=0}^{\infty}\alpha_1^{(s)}P_s(x), \tag{7}$$

where $x = \cos\Theta$ and $\alpha_1^{(s)}$ is the expansion coefficient of order s. The δ-fit method approximates this phase function with a limited number of coefficients N that are the result of fitting in the form:

$$\hat{\mathcal{P}}_N(x) = \sum_{s=0}^{N}\hat{\alpha}_1^{(s)}P_s(x), \tag{8}$$

where a hat indicates that the quantity is a result of fitting.

The standard least squares method is employed to minimize the sum of squared differences between 1 and the ratio of the reconstructed phase function $\hat{\mathcal{P}}_N(x)$ to the original phase function $\mathcal{P}(x)$:

$$\varepsilon = \sum_{i=1}^{m}\left(\frac{\hat{\mathcal{P}}_N(x_i)}{\mathcal{P}(x_i)} - 1\right)^2 w_i, \tag{9}$$

where w_i is the step-function weight defined by the truncation angle Θ_{trc}.

$$w_i = \begin{cases} 1, & x < \cos\Theta_{trc} \\ 0, & x \geq \cos\Theta_{trc.} \end{cases} \tag{10}$$

Linear least square fitting based on Eq. (9) by singular vector decomposition yields a set of coefficients $\hat{\alpha}_1^{(s)}$. Once a set of coefficients is obtained, the similarity transformation of the radiative transfer equation is utilized to satisfy the normalization condition of the phase function. Specifically, the following scaling adjustments are applied:

$$1-f = \hat{\alpha}_1^{(0)}, \tag{11}$$

$$\hat{\alpha}_1^{(s)*} = \frac{\hat{\alpha}_1^{(s)}}{1-f}. \tag{12}$$

The scaling factor (truncation factor) f and fitted, scaled expansion coefficients $\hat{\alpha}_1^{(s)*}$ are used in an arbitrary solver of the scaled radiative transfer equation.

The fitting process in Eq. (9) can be rewritten as a linear least square fitting with weight inversely proportional to the variance in the tabulated phase function in the form:

$$\varepsilon/{\varepsilon_1}^2 = \sum_{i=1}^{m}\left(\hat{\mathcal{P}}_N(x_i) - \mathcal{P}(x_i)\right)^2\frac{w_i}{(\varepsilon_1\mathcal{P}(x_i))^2}, \tag{13}$$

where ε_1 is the precision of the phase function. Note that $(\varepsilon_1\mathcal{P}(x_i))^2$ is the variance of the phase function at the ith data point. The variance is due to the numerical implementation of light scattering calculations and the numerical representation of a phase function. The precision, ε_1, is chosen as the largest value among machine precision, table precision, and the relative error in the numerical algorithm.

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