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Groups in the radiative transfer theory

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ABSTRACT

The paper presents a group-theoretical description of radiation transfer in inhomogeneous and multi-component atmospheres with the plane-parallel geometry. It summarizes and generalizes the results obtained recently by the author for some standard transfer problems of astrophysical interest with allowance of the angle and frequency distributions of the radiation field. We introduce the concept of composition groups for media with different optical and physical properties. Group representations are derived for two possible cases of illumination of a composite finite atmosphere. An algorithm for determining the reflectance and transmittance of inhomogeneous and multi-component atmospheres is described. The group theory is applied also to determining the field of radiation inside an inhomogeneous atmosphere. The concept of a group of optical depth translations is introduced. The developed theory is illustrated with the problem of radiation diffusion with partial frequency distribution assuming that the inhomogeneity is due to depth-variation of the scattering coefficient. It is shown that once reflectance and transmittance of a medium are determined, the internal field of radiation in the source-free atmosphere is found without solving any new equations. The transfer problems for a semi-infinite atmosphere and an atmosphere with internal sources of energy are discussed. The developed theory allows to derive summation laws for the mean number of scattering events underwent by the photons in the course of diffusion in the atmosphere.

1. Introduction

Interpretation of the line spectra of cosmic objects often encounters principal difficulties due to both the inhomogeneity of radiating media and the multiple scattering of the line radiation occurring with the angle and frequency changes [1,2]. Of particular interest is the special case of the media with a fine multi-component structure taking into account that the detailed observation and investigation of structural patterns are within the limits of abilities of the present-day powerful instruments which provide a rich observational material. A lot of weighty examples of them can be presented from various branches of astrophysics. Strictly speaking, the real line-radiating media are always inhomogeneous, and the commonly used assumption of homogeneity is made to simplify the theoretical treatment of problems allowing then to find several averaged in some sense parameters of radiating volumes. This leads to pressure to develop a more general approach to problems of radiative transfer through inhomogeneous and composite media and elaborate numerical methods better adapted to capabilities of the modern electronic machines.

The basic goal of this paper is to propose a relatively simple an user-oriented technique aimed at determining and interpreting the observable and measurable features of the line-radiating media. It assumes application of the group theory methods to solution of standard radiation transfer problems under general assumptions on the elementary act of scattering and optical properties of the medium.

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The theory we put forward can be regarded as a further extension of the layers adding method proposed by Ambartsumian [3–5] for one-dimensional homogeneous media and generalized in [6–8] to encompass inhomogeneous media. We remind that the method establishes summation laws for global optical properties of absorbing and scattering media (reflectance and transmittance), which expresses these properties of the combined medium in terms of similar properties of its components. It is noteworthy the limiting case of the method when optical thickness of one of the added components tends to zero. This provides finding the global optical characteristics of a medium simultaneously for a family of media of different thicknesses. This branch of theory was developed by Bellman and his co-authors (see e.g., [9,10]) and is known as 'invariant imbedding'. Finally, the specific case, in which the addition procedure is subject to the condition of invariability of the optical thickness of a medium, was treated by Ambartsumian in [11] (see also [1,4]) and was called 'invariance principle'. This makes it possible to derive the outgoing intensities without primary determining the field of radiation inside an atmosphere. As it was shown in [8,12,13], the equations resulting from the principle are manifestation of the laws ensuing from the symmetry properties of homogeneous transfer problems with respect to optical depth translations.

The approach to which we adhere in our recent papers (see e.g. [7,8,14,15]) including the present one assumes preliminary determination of the global optical properties of the medium. It leads to comparatively easy solvable initial-value problems and, as we shall see, simplifies the problem of finding the field of radiation inside a medium. In addition, the approach serves as a basis for applying methods of the group theory in describing the radiation transfer phenomenon inside an absorbing and scattering atmosphere [16–18].

The outline of the paper is as follows. In the opening Section 2, we introduce the concept of composition groups describing the addition of absorbing and scattering media. Next section presents the basic discrete quantities which determine the optical properties of inhomogeneous atmosphere. The representations of composition groups are derived. In Section 4, the approach is exemplified on the standard source-free problem for partial redistribution of radiation over frequencies. The multi-component atmosphere is considered in Section 5. Further, we focus the attention to the problem of determining the field of radiation inside a medium. The concept of groups of the depth translations is introduced. It is shown that, once the global optical properties of an atmosphere are found, the internal field of radiation is determined without solving any new equations. The next two sections treat the transfer problems for an atmosphere containing energy sources and the semi-infinite atmosphere. In Section 9, the developed method is applied to find the statistical mean quantities which describe the radiation diffusion in the medium. The results obtained in the paper are discussed in the final section.

2. Composition groups

We start with considering the amalgamation procedure of the plane-parallel absorbing and scattering inhomogeneous media. It is assumed that the added components do not contain primary energy sources and are allowed to differ one from the other not only by optical thicknesses, but also by the nature of inhomogeneity. Under inhomogeneity we mean that each of the physical parameters specifying the elementary event of scattering or physical state of the medium may vary with depth. Of them we note the profile of the absorption coefficient, the quantum scattering (or destruction) coefficient, Voigt's parameter, indicatrix, the frequency redistribution function, the Stokes parameters in the case of polarized radiation, the correlation length for turbulent media, and so on. The operator-matrix approach developed in the paper involves a broad class of three-dimensional and frequency-dependent problems. However, in illustrating the approach, we restrict ourselves by treating the 1D transfer problem for the case of partial redistribution over frequencies by assuming that the only variable parameter is the photon 'survival' probability during the elementary act of scattering, or briefly, the scattering coefficient denoted by λ.

Now we introduce the concept of a composition or transformation of scattering and absorbing inhomogeneous media, which refers to the addition of a new medium to the initial one. The transformations induced in this way form a group if under the group product (binary operation) one takes the resultant of two successive transformations. It is remarkable that this definition does not specify the nature of inhomogeneity of added media. It is easily seen that all the required conditions for forming a group are satisfied. In particular, the role of the unit element is played by the identity transformation, which leaves the initial medium unchanged, and the inverse element is the transformation which reverses the effect of the already performed transformation. The associativity of the group product is obvious. We refer to this group of transformations as the GN(2,C) group, which evidently is not commutative. As a result of such compositions, one can construct different atmospheres composed of inhomogeneous components. Of special interest is one of subgroups of the introduced group which refer to the case when the added media are homogeneous. The components of such a composite atmosphere may differ each from other not only by optical thicknesses but also by any characteristics of the radiation diffusion in them. Such groups, referred nominally to as GNH(2,C), are two-, three- and multi-parameter dependent on the number of parameters changing in passing from one component to another. These groups are infinite and non-commutative. They can serve as archetypes for a number of real radiating media of astrophysical importance. Finally, of independent interest is the narrower subgroup of the introduced two groups which involves compositions of homogeneous media with identical physical properties but, in general, of different optical thicknesses. These compositions obviously yield homogeneous medium. This one-parameter group, we call it GH(2,C), is infinite and commutative, i.e., Abelian [15]. It becomes continuous when the only parameter, optical thickness, varies continuously.