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On the linear properties of the nonlinear radiative transfer problem

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ABSTRACT

In this report, we further expose the assertions made in nonlinear problem of reflection/transmission of radiation from a scattering/absorbing one-dimensional anisotropic medium of finite geometrical thickness, when both of its boundaries are illuminated by intense monochromatic radiative beams. The new conceptual element of well-defined, so-called, *linear images* is noteworthy. They admit a probabilistic interpretation. In the framework of nonlinear problem of reflection/transmission of radiation, we derive solution which is similar to linear case. That is, the solution is reduced to the linear combination of linear images. By virtue of the physical meaning, these functions describe the reflectivity and transmittance of the medium for a single photon or their beam of unit intensity, incident on one of the boundaries of the layer. Thereby the medium in real regime is still under the bilateral illumination by external exciting radiation of arbitrary intensity. To determine the linear images, we exploit three well known methods of (i) adding of layers, (ii) its limiting form, described by differential equations of invariant imbedding, and (iii) a transition to the, so-called, functional equations of the "Ambartsumyan's complete invariance".

1. Introduction

From its historical development, up to current interests much remarkably efforts have been made in the study of linear problem of radiative transfer in the scattering/absorbing medium. This has evoked an extensive scientific literature (e.g. [1–8]). Meanwhile, the analytical and numerical/analytical, as well as purely numerical aspects of their analysis are widely developed. In nonlinear case, however, the calculations of radiation field have still carried out mostly numerically – in the "forehead" manner [9,10]. Because of a mathematical complexity, nonlinear transfer problem seems doomed and remains poorly understood from the standpoint of both analytical and numerical/analytical methods (e.g. [11–30]). In the linear problem of multiple interactions of radiation with matter, usually, only the change of initial characteristics of the radiation field is taken into accounted. Whereas the scattering/absorbing properties of the medium itself (i.e. the radiative properties of the elementary volume of the given medium) are specified in advance. These properties have not changed in a single act of interaction of radiation with matter. In contrast, a crucial caveat involved in nonlinear problem is as follows: under multiple interactions of radiation with matter, the resulting characteristics of the radiation field and the physical state of the medium form each other reciprocally, in self-consistent manner.

With this perspective in sight, a major goal of this report is to supplement and further simplify the methods of analytical and numerical/analytical solutions of nonlinear problems in radiative transfer by exploring some plausible linear properties. We address the simple monochromatic problem of one-dimensional scattering/absorbing anisotropic medium of finite geometrical thickness, when both of its boundaries are illuminated by intense

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monochromatic radiative beams. We proceed according to the following structure. Sections 2–5 recount some of the highlights, as a guiding principle, behind the linear and nonlinear transfer problems for the benefit of the reader to make the rest of paper understandable. We briefly review the key points of the problems, provide an analysis aimed at clarifying the current situation and brought all the formulas necessary for further discussion. Although some key theoretical ideas were introduced with a satisfactory substantiation, we have also attempted to maintain a balance being overly detailed and overly schematic. Whereas, the three well known methods of the exact solution of the nonlinear problem of reflection and transmission (PRT) of radiation by the one-dimensional anisotropic medium of finite geometrical thickness are outlined. This includes (i) the method of adding of layers and (ii) its limiting form, described by differential equations of invariant imbedding, and (iii) a transition to the so-called functional equations of the "Ambartsumyan's complete invariance" (ACI). Without care of the historical justice and authenticity, it should be emphasized that even though being among the most significant advances in transfer theory, all these proposals rather stem from the invariance principle [27–29]. Conceptually and techniquewise the invariance principle is versatile and powerful. Each of the mentioned above proposal has its own advantages and difficulties. In concrete situation, the interests of the problem in quest and the premises of its implications define the feasibility of choice of one of them. In practice, however, an essential insight is often gained by incorporating with all the different features of three methods, as alluded to above. For brevity reasons, we shall not attempt a history of these three proposals in general, but only of those that seem most relevant to the particular theory of this paper. We will refrain from providing lengthy details, and deliberately forebear from presumption of actual discussion of transfer theory, which is almost a routine job that rather requires extensive and careful analysis. This can be found in most classic texts on radiative transfer. The interested reader, therefore, is invited to consult the original papers. Besides, in the literature there are various, more or less ad hoc, lists of criteria of reasonableness of the approximate solutions of nonlinear problem. However, finding an appropriate one has proven to be surprisingly difficult, and currently no single approach has been uniquely accepted as the convincing one, which could give a consistent solution of nonlinear transfer theory. Our interest, therefore, is to further expose here the assertions made in the three proposals above via the unifying and simplifying them novel method.

1.1. Rational

A new physical property of the multiple scattering processes in nonlinear case indeed is revealed in Section 6. Continuing on our quest, we next lay forth the foundation of the most important concepts of so-called *linear images*, which have a heuristic value. Envisaging that the solution of nonlinear PRT can be reduced to their simple linear combination, we have promoted these concepts to the status of robust structures. Intuitively speaking, it may not be too unreasonable to conceive such an extension. The concepts of linear images are a bold assumption in its own right, and they, hopefully, may become of eminent physical significance for investigation of nonlinear transfer problem. So, one of our purposes is to motivate and justify these concepts, and by their means to circumvent the obstacles existing in nonlinear problem. To find the linear images, in Section 7 we derive corresponding formulas for adding of layers, which have a structure similar to those of linear case. There is another line of reasoning which supports the side of these concepts. The unifying feature of the suggested method allows to construct the algorithm for step-by-step tackling of nonlinear PRT in terms of linear images. The second method, given in Section 8, of solving nonlinear PRT by means of linear images, rather implies a sequential transition to the limit of elementary layer in the formula of adding of two layers. Continuing along this line, the key radiation characteristics of a single elementary layer are explicitly determined in Section 9. Consequently, a complete set of the equations of invariant imbedding, with corresponding initial conditions, is obtained in Section 10. The initial conditions include some auxiliary functions. For their determination, we formulate all the necessary nonlinear (Section 11), as well as linear (Section 12), particular PRT. In this context, perhaps, the third way of deriving the linear images is the transition from the complete set of equations of invariant imbedding to the functional equations of ACI. This is demonstrated in Section 13. Finally, in an illustration of the point at issue, a simple instructive example is given in Section 14. The concluding remarks are brought in Section 15. This is not a final report on a closed subject, but it is hoped that suggested novel view point will serve as useful introduction and that it will thereby add the knowledge on the role of the linear properties in nonlinear problems of radiative transfer.

2. Equation of radiative transfer

In the framework of radiative transfer theory, it is almost a matter of routine, to proceed from the classical Boltzmann equation, so-called equation of radiative transfer, written for the particles of "photon gas"

$$\frac{\partial I_L^{\pm}}{\partial l} = \pm \alpha^{\pm}\left(I_L^{+}, I_L^{-}\right), \tag{1}$$

where $I_L^{\pm} \equiv I_L^{\pm}(l;x,y)$ is the intensity of radiation at a depth l of one-dimensional scattering/absorbing anisotropic medium of finite geometrical thickness L. It was illuminated from the left ($l=0$) and right ($l=L$) boundaries by intense radiation beams with intensities x and y, respectively (positive direction is selected along the increase of layer thickness). Whereas, $0 \le x, y < \infty, L \ge 0$, and $0 \le l \le L$, and $\alpha^{\pm}\left(I_L^{+}, I_L^{-}\right)$ is the well-known integral of collisions of the problem. For the formulation of two-point boundary value problem, Eq. (1) is supplemented by the boundary conditions

$$I_L^{+}(0;x,y) = x, \quad I_L^{-}(L;x,y) = y, \tag{2}$$