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Radiance and polarization in the diffusion region with an arbitrary scattering phase matrix

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ABSTRACT

Radiance and polarization patterns in an optically deep region, the so-called diffusion region or asymptotic region, of a homogeneous atmosphere or ocean, depend mainly on the scattering phase matrix and the single-scattering albedo of the medium. The radiance and polarization properties in the diffusion region for an arbitrary scattering phase matrix can be obtained in terms of a series of the generalized spherical functions. The number of terms is closely related to the single-scattering albedo of the medium. If the medium is conservative, the radiance is isotropic in conjunction with no polarization. If the single-scattering albedo is close to 1, several terms are sufficient to obtain the patterns, in which the degree of polarization feature is less than 1%. If the medium is highly absorptive, more expansion terms are required to obtain the diffusion patterns. The examples of simulated radiance and polarization patterns for Rayleigh scattering, Henyey–Greenstein–Rayleigh scattering, and haze L and cloud C1 scattering, defined by Deirmendjian, are calculated.

1. Introduction

The domain optically deep in a homogenous atmosphere or ocean, far from the top and bottom boundaries, is defined as the diffusion region or the asymptotic region. If the medium is conservative, the photons have made so many scatterings that they lose "memories" of the direction from which they came deep into the medium and hence the radiance eventually becomes isotropic in conjunction with no polarization. If the medium is non-conservative, the radiance and polarization in the diffusion region, which will be proportional to $\exp(-k\tau)$, where k and τ are the diffusion exponent and the optical depth, depend primarily on the inherent optical properties of the medium, that is, the scattering phase matrix and the single-scattering albedo, but not on the initial conditions. Moreover, the patterns are independent of the azimuthal angle.

Radiance and polarization properties in the diffusion region have been extensively studied from both theoretical and experimental perspectives [1–14]. The analytical radiance and polarization patterns for Rayleigh scattering have been reported, and patterns for haze L and cloud C1 scattering have been numerically solved using the Lobatto quadrature by Kattawar and Plass [15]. The radiance pattern in the diffusion region is expressed in terms of Legendre polynomials by van de Hulst [16]. As reported by Kattawar and Plass [15], the error of the radiance derived from the scalar theory can be as much as 16.4%. Subsequently, polarization must be included to obtain an accurate result. The radiation field of polarized light of a semi-infinite atmosphere has been thoroughly studied in terms of the generalized spherical functions (GSFs) [17–19] by Domke and de Rooij [20,21]. In this study, the single

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scattering albedo ω_0 is factored out instead of incorporated in the phase matrix and the diffusion exponent k is given for convenience in the calculation. The characteristic equation is explicitly written as a polynomial of the single scattering albedo ω_0. Moreover, the patterns for Rayleigh scattering, Henyey–Greenstein–Rayleigh scattering, and haze L and cloud C1 scattering defined by Deirmendjian [22] are simulated, which are validated in terms of the adding-doubling method in the limit of an extremely large optical depth.

2. Solution for radiance and polarization with an arbitrary scattering matrix

2.1. Expansion using generalized spherical functions

The Stokes parameters I, Q, U, V are independent of azimuthal angle in the diffusion region, and U and V are zero. In the diffusion region, if the medium is non-conservative, i.e., the single scattering albedo $\omega_0 < 1$, the integral equation of the radiance and polarization patterns $P(\mu)$ and $T(\mu)$ can be written as [8,15,16]

$$(1+k\mu)\begin{pmatrix} P(\mu) \\ T(\mu) \end{pmatrix} = \frac{\omega_0}{2}\int_{-1}^{1} \mathbf{H}(\mu,\mu')\begin{pmatrix} P(\mu') \\ T(\mu') \end{pmatrix} d\mu', \tag{1}$$

where k is the diffusion exponent, μ is the cosine of the zenith angle, which is positive for upwelling directions and negative for downwelling directions, $P(\mu)$ and $T(\mu)$ are the corresponding diffusion patterns of the Stokes elements I and Q, and

$$\mathbf{H}(\mu,\mu') = \sum_{n=0}^{\infty} \mathbf{P}_0^n(\mu)\mathbf{S}^n\mathbf{P}_0^n(\mu'), \tag{2}$$

where

$$\mathbf{P}_0^n(\mu) = \begin{pmatrix} P_{0,0}^n(\mu) & 0 \\ 0 & P_{0,2}^n(\mu) \end{pmatrix}, \tag{3}$$

$$\mathbf{S}^n = \begin{pmatrix} \alpha_1^n & \beta_1^n \\ \beta_1^n & \alpha_2^n \end{pmatrix} \tag{4}$$

α_1^l, α_2^l, and β_1^l are the expansion coefficients of a scattering matrix $F(\mu)$ in terms of GSFs, where

$$F(\mu) = \begin{pmatrix} a_1(\mu) & b_1(\mu) & 0 & 0 \\ b_1(\mu) & a_2(\mu) & 0 & 0 \\ 0 & 0 & a_3(\mu) & b_2(\mu) \\ 0 & 0 & -b_2(\mu) & a_4(\mu) \end{pmatrix}, \tag{5}$$

and

$$\begin{cases} \alpha_1^n = (n+\frac{1}{2})\int_{-1}^{1} d\mu\left(P_{0,0}^n(\mu)a_1(\mu)\right) \\ 2\alpha_2^n = (n+\frac{1}{2})\int_{-1}^{1} d\mu\left[\left(P_{2,2}^n(\mu)+P_{2,-2}^n(\mu)\right)a_2(\mu)+\left(P_{2,2}^n(\mu)-P_{2,-2}^n(\mu)\right)a_3(\mu)\right] \\ \beta_1^n = (n+\frac{1}{2})\int_{-1}^{1} d\mu\left(P_{0,2}^n(\mu)b_1(\mu)\right) \end{cases}. \tag{6}$$

The $P_{0,0}^n$, $P_{0,2}^n$, $P_{2,2}^n$, and $P_{2,-2}^n$ terms are the GSFs, whose initial functions and recurrence relations can be found in [19].

The diffusion patterns $P(\mu)$ and $T(\mu)$ can also be expanded in terms of the GSFs

$$\begin{cases} P(\mu) = \sum_{n=0}^{\infty}(2n+1)i_n P_{0,0}^n(\mu) \\ T(\mu) = \sum_{n=2}^{\infty}(2n+1)q_n P_{0,2}^n(\mu) \end{cases}, \tag{7}$$

and then

$$\begin{cases} (1+k\mu)P(\mu) = \sum_{n=0}^{\infty} a_n P_{0,0}^n(\mu) \\ (1+k\mu)T(\mu) = \sum_{n=2}^{\infty} b_n P_{0,2}^n(\mu) \end{cases}, \tag{8}$$

where

$$a_n = \begin{cases} i_0 + k i_1, \quad n=0 \\ (2n+1)i_n + k\cdot n i_{n-1} + k\cdot(n+1)i_{n+1}, \quad n\ge 1 \end{cases}, \tag{9}$$

and

$$b_n = \begin{cases} 5q_2 + k\cdot\sqrt{5}q_3, n=2 \\ (2n+1)q_n + k\cdot\sqrt{n^2-4}\,q_{n-1} + k\cdot\sqrt{(n+1)^2-4}\,q_{n+1}, n\ge 3 \end{cases}. \tag{10}$$

Upon substituting Eqs. (7)–(10) into Eq. (1), and using the orthogonal properties of the GSFs we arrive at the following recurrence relationships

$$i_1 = h_0\gamma i_0, 2i_2 + i_0 = h_1\gamma i_1, \sqrt{5}q_3 = f_2\gamma q_2 + g_2\gamma i_2, \tag{11}$$

$$\begin{cases} (n+1)i_{n+1} + n i_{n-1} = h_n\gamma i_n + g_n\gamma q_n, \quad n\ge 2 \\ \sqrt{(n+1)^2-4}\,q_{n+1} + \sqrt{n^2-4}\,q_{n-1} = f_n\gamma q_n + g_n\gamma i_n, \quad n\ge 3 \end{cases}, \tag{12}$$

where

$$\begin{aligned} h_n &= \omega_0\alpha_1^n - (2n+1), \\ f_n &= \omega_0\alpha_2^n - (2n+1), \\ g_n &= \omega_0\beta_1^n, \gamma = 1/k. \end{aligned} \tag{13}$$

2.2. Truncation and characteristic equation

The diffusion exponent k and the single-scattering albedo ω_0 are a coupled pair. A characteristic equation is necessary to find ω_0 if k is known, and vice versa. This study assumes k is known for convenience. In the expansions of the scattering phase matrix, a finite order N is reasonably assumed, or, $\alpha_1^n = \alpha_2^n = \beta_1^n = 0$ if $n > N$ Moreover, the expansion coefficients $a_n = b_n = 0$ if $n > N$ The recurrence equations are decoupled for $n > N$ as

$$\begin{cases} (n+1)i_{n+1} + n i_{n-1} = (2n+1)(-\gamma)i_n \\ \sqrt{(n+1)^2-4}\,q_{n+1} + \sqrt{n^2-4}\,q_{n-1} = (2n+1)(-\gamma)q_n \end{cases}. \tag{14}$$

The recurrence equations are almost the same as for the GSFs $P_{0,0}^n$ and $P_{0,2}^n$, except that the argument $-\gamma$ here is smaller than -1 while $-1 \le \mu \le 1$ The expansions in Eq. (7) are valid only if $i_n \to 0$ and $q_n \to 0$ when $n \to \infty$ First,