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Light scattering by ice crystals of cirrus clouds: comparison of the physical optics methods

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ABSTRACT

The physical optics approximations are derived from the Maxwell equations. The scattered field equations by Kirchhoff, Stratton-Chu, Kottler and Franz are compared and discussed. It is shown that in the case of faceted particles, these equations reduce to a sum of the diffraction integrals, where every diffraction integral is associated with one plane–parallel optical beam leaving a particle facet. In the far zone, these diffraction integrals correspond to the Fraunhofer diffraction patterns. The paper discusses the E-, M- and (E, M)-diffraction theories as applied to ice crystals of cirrus clouds. The comparison to the exact solution obtained by the discontinuous Galerkin time domain method shows that the Kirchhoff diffraction theory is preferable.

1. Introduction

The problem of light scattering by ice crystals of cirrus clouds is a challenging problem of atmospheric optics because of the strong impact of the radiation scattered by the clouds on the Earth's climate [1–3]. However, the problem of light scattering by a single typical crystal of cirrus clouds has not been satisfactorily solved yet. Indeed, the so-called exact methods that are, in fact, various numerical solutions of the corresponding Maxwell equations become too computationally costly if the particle size exceeds ~30 wavelengths of the incident light. Among the methods, the finite dimension time-domain (FDTD) method [4,5], the discrete dipole approximation (DDA) [6,7], the T-matrix method [8,9] and the discontinuous Galerkin time domain method (DGTD) [10,11] are mostly known. Recently, some generalized methods like the pseudo-spectral time domain method (PSDT) [12,13] and the invariant imbedding T-matrix method (II-TM) [14,15] have been developed to overcome the limit of (size/wavelength) < 30. However, all the exact methods are not applicable yet to calculate lidar signals scattered from cirrus clouds.

Since the cirrus ice crystals size ranges from a few microns to several millimeters, various approximations based on the geometric-optics approach to the scattered field are widely used for practical applications. The evolution of the above methods has a long history; a recent survey of these works together with comprehensive references can be found, e.g., in [16]. At the same time, it is the diffraction phenomenon that takes the wave properties of light into account in an obvious and simple way. Therefore, the concept of diffraction distinguishes between the physical and geometric optics. In the papers presented in the survey [16] a reader can find a complicated confusion of the concepts of both geometric and physical optics.

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Recently, we have suggested a simple classification of the geometric-optics and physical-optics approximations [17] for the problem of light scattering by crystals which are much larger than the incident wavelength. The principal assumption is that the scattered field either inside a particle or on its surface should be described only within the framework of geometric optics. Here the geometric-optics field can be found using three kinds of algorithms. The first kind of algorithms is the conventional ray-tracing procedure [18,19] dealing with separate geometric-optics rays. In the second kind of the algorithms, the geometric-optics rays are combined into narrow plane–parallel beams or tubes of given transverse shapes [20,21]. In the case of flat crystal facets plane–parallel beams appear. Finally, the third kind of algorithms operates with transversely wide plane–parallel beams which are initiated by the whole illuminated face [22,23]. Of course, all the algorithms should result in the same value of the geometric-optics field. It is worth noting that at present two algorithms are open for free access. They are the ray-tracing code by Macke [19] and our beam-splitting code [22].

While the geometric-optics field is found on the particle surface, the next step is to calculate a scattered field in the far zone, where scattered radiation is usually detected. If every geometric-optics ray of the scattered field is assumed to propagate directly to the far zone, we obtain the geometric-optics solution of the scattering problem. The above solution is not satisfactory because it does not take the wave properties of light into account and, consequently, it does not satisfy experimental data. Therefore, to improve the solution, some authors have added the diffraction phenomenon artificially, especially in the forward scattering direction [18,19,24]. Instead of this, there is a strict mathematical procedure of mapping the scattered field from particle surface to the far zone. This is the exact integral equation following from the Maxwell equations [25]. This equation determines the scattered field at any point of the far zone through an integral of the scattered field over the particle surface. Note that there is a similar integral over the particle volume as well omitted here issue for brevity [26].

Substitution of the geometric-optics field obtained on the crystal surface as a set of plane–parallel beams reduces the problem of finding the far-zone scattered field to a sum of the diffraction patterns produced by every beam that is discussed in detail below. Here every beam produces the Fraunhofer diffraction pattern formed by an aperture in a flat screen, with the shape of the aperture coinciding with the beam shape. Thus, in the case of faceted particles, the scattering problem and the diffraction problem become exactly equivalent in terms of the physical optics approximation. As is known, the diffraction at an aperture of the size essentially exceeding the incident wavelength can be described by different diffraction integrals, which are associated with the Kirchhoff and Rayleigh–Sommerfeld approximations in the case of scalar waves. Therefore, there arises a problem, which of these diffraction integrals is more suitable for the physical-optics approximation?

This paper is organized as follows. In Section 2, the physical optics approximation is derived from the Maxwell equations. The different forms of the scattered field equations are discussed in Section 3. Section 4 deals with the equivalence of the scattering and diffraction theories in terms of the physical optics approximation. The numerical comparison of different physical optics approximations is presented in Section 5.

2. Theoretical basis of the physical optics approximation

Let us start with the general problem of monochromatic wave scattering by a nonmagnetic particle in vacuum (magnetic permeability outside the particle is equal to 1). In the case of a homogeneous dielectric isotropic particle, the well-known Maxwell equations become the following vector equation for the time independent part of the electric field $\mathbf{E}(\mathbf{r})$ [27–29]

$$\nabla\times\nabla\times\mathbf{E}(\mathbf{r})-k^2\mathbf{E}(\mathbf{r})=\mathbf{F}_e(\mathbf{r}), \tag{1}$$

where $\nabla\times$ is the curl operator, $\mathbf{r}=(x,y,z)$ is an arbitrary point in space, $k=2\pi/\lambda$ is the wave number, λ is the wavelength,

$$\mathbf{F}_e(\mathbf{r})=\begin{cases} k^2\left[1-n^2\right]\mathbf{E}(\mathbf{r}), & \text{inside the particle} \\ \mathbf{j}_{ext}(\mathbf{r})\cdot 4\pi k/c, & \text{outside the particle} \end{cases} \tag{2}$$

where n is the refractive index, c is the electromagnetic constant, $\mathbf{j}_{ext}$ is the electric current density that represents the incident wave.

To solve Eq. (1), we introduce the dyadic Green function [30]

$$\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')=\left(\frac{\nabla\nabla}{k^2}+\overline{\overline{\mathbf{U}}}\right)\frac{e^{ik|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}, \tag{3}$$

which obeys the condition [27,31]

$$\nabla\times\nabla\times\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')-k^2\overline{\overline{\mathbf{G}}}(\mathbf{r},\mathbf{r}')=\delta(\mathbf{r}-\mathbf{r}')\overline{\overline{\mathbf{U}}}, \tag{4}$$

where $\overline{\overline{\mathbf{U}}}$ is the unit dyadic.

Bearing in mind the vector–dyadic Green theorem [31]

$$\int_V\left(\mathbf{P}\cdot\nabla\times\nabla\times\overline{\overline{\mathbf{Q}}}-\overline{\overline{\mathbf{Q}}}\cdot\nabla\times\nabla\times\mathbf{P}\right)dv = -\oint_S\left(\mathbf{P}\times\nabla\times\overline{\overline{\mathbf{Q}}}+(\nabla\times\mathbf{P})\times\overline{\overline{\mathbf{Q}}}\right)\cdot\mathbf{n}ds \tag{5}$$

and letting $\mathbf{P}=\mathbf{E}$, $\overline{\overline{\mathbf{Q}}}=\overline{\overline{\mathbf{G}}}$, we obtain:

$$\int_V\left(\mathbf{E}\cdot\nabla'\times\nabla'\times\overline{\overline{\mathbf{G}}}-\overline{\overline{\mathbf{G}}}\cdot\nabla'\times\nabla'\times\mathbf{E}\right)dv' = -\oint_S\left(\mathbf{E}\times\nabla'\times\overline{\overline{\mathbf{G}}}+(\nabla'\times\mathbf{E})\times\overline{\overline{\mathbf{G}}}\right)\cdot\mathbf{n}ds', \tag{6}$$

where $\mathbf{n}$ is the outward normal and the symbol $'$ means that the integration and differentiation are taken over $\mathbf{r}'$.

Eq. (6) allows us to express the unknown electric field $\mathbf{E}$ at the arbitrary point $\mathbf{r}$ of the free space V as an integral over the external particle surface S_p^+ (see Fig. 1).