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A comparison of specularly reflective boundary conditions and rotationally invariant formulations for Discrete Ordinate Methods in axisymmetric geometries

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ABSTRACT

In simulations of periodic or symmetric geometries, computational domains are reduced by imaginary boundaries that exploit the symmetry conditions. Two boundary conditions are proposed for Discrete Ordinate Methods to solve axisymmetric radiation problems. Firstly, a specularly reflective boundary condition similar to that is used in Photon Monte Carlo methods is developed for Discrete Ordinate Methods. Secondly, the rotational invariant formulation is revisited for axisymmetric wedge geometries. Correspondingly, a new rotationally invariant boundary condition specially designed for axisymmetric problems on wedge shape is proposed to enforce the rotational invariance properties possessed by the radiative transfer equation (RTE) but violated by three-dimensional conventional Discrete Ordinate Methods. Both boundary conditions have the advantage that the discretization and linear equation solution procedures of conventional three-dimensional DOM are not affected by changing to a reduced geometry. Consistency, accuracy and efficiency of the new boundary conditions are demonstrated by multiple numerical examples involving periodic symmetry and axisymmetry. A comparison between specularly reflective boundary conditions and the rotationally invariant formulation shows that the latter offers several advantages for wedge geometries. In other symmetry conditions, when the rotational invariant formulation is not applicable, specular reflective boundary conditions are still effective.

1. Introduction

Periodic and symmetric geometries are frequently encountered in engineering applications. Examples of such geometries include periodic structures, plane symmetry and axisymmetry. In the simulations of these problems, flow variables are solved for only a section of the domain. The computational domain is separated from the rest by imaginary boundaries, upon which symmetric constraints as opposed to physical conditions are applied.

While symmetry constraints may be easily expressed into mathematical formulas for scalar and vector fields as frequently performed in CFD simulations, they present challenges for radiative intensities. This is because the radiative intensities are functions of both spatial and directional coordinates [1]. Plane and rotational symmetric conditions are complicated by the additional directional variations. As an example, scalar flow variables in an axisymmetric problem are only functions of radial and axial

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locations. In flow solvers designed for three-dimensional coordinates only, a wedge geometry that has only one layer of cells in the azimuthal direction may be employed. Symmetry conditions are imposed on the imaginary wedge planes. However, even in these axisymmetric problems, the radiative intensities are three dimensional, i.e., light propagation is not confined to the plane formed by the radial and axial direction vectors. Such issues are also observed in Photon Monte Carlo (PMC) methods for radiative transfer. In PMC methods, the wedge planes may be treated as specularly reflective boundaries, i.e., a ray hitting the wedge plane is reflected back into the computational domain like hitting a perfect mirror.

When applying Discrete Ordinate Methods [1,2] to axisymmetric simulations, three methods are typically employed. In the first method, the radiative transfer equation (RTE) is solved for ordinate intensities in a full three-dimensional axisymmetric domain, as in Refs. [3–5]. This method cannot take advantage of symmetry to reduce computational cost. In the second method, the RTE is formulated in cylindrical coordinates directly as in Refs. [6–11]. However, this approach requires dedicated implementations of DOM equations for two-dimensional problems. In the third method, a reduced mesh based on symmetry is used to discretize the RTE formulated for Cartesian coordinates [12,13]. Axisymmetry is enforced by special treatment of the wedge boundaries. This method simplifies the implementation of the RTE, because the same equations, discretization and linear equation solution methods are used for both reduced and full geometries. However, the resulting new wedge boundaries require different treatment from that is employed for scalar or vector partial differential equations.

In this work, after a short review of the Discrete Ordinate Methods and their Finite Volume discretizations in Sections 2.1 and 2.2, a specularly reflective boundary condition is proposed in Section 2.3 for Discrete Ordinate Methods in recognition of the fact that the ordinates have similarities to rays in Photon Monte Carlo methods. The specularly reflective boundary conditions are expressed as mathematical boundary conditions for the first-order partial differential equations that govern spatial variations of each ordinate depending on the spatial relationship between the surface normal and the ordinate directions. It is shown that the implementation of the specularly reflective boundary condition varies with the finite volume interpolation scheme for the face values. The specularly reflective boundary condition can be applied to both periodic and axisymmetric reduced geometries. In Section 2.4, a rotationally invariant formulation is derived for axisymmetric problems in wedge geometries. Its implementation results in a specially designed boundary condition for the wedge boundaries (Section 2.5), so that the remaining Finite Volume discretization of the DOM equations is unchanged. Similarities and differences of the two boundary conditions are highlighted in Section 2.6. Several tests are performed in Section 3 to further examine the consistency, accuracy and efficiency of the two boundary conditions in reduced geometries.

2. Theoretical background

2.1. Discrete ordinate methods

In this section, the Discrete Ordinate Methods for solving the radiative transfer equation are briefly reviewed. The presentation only focuses on necessary content for the new development as opposed to completeness. We will limit the theoretical discussion to non-scattering participating media, because the treatment of scattering is not essential to this work and does not impose new technical difficulties. Readers are referred to Ref. [1] for more comprehensive discussions of the DOM and Ref. [2] for the review of recent developments.

The radiative transfer equation (RTE) for a radiatively participating gray medium with emission and absorption is a first order differential equation:

$$\hat{s} \cdot \nabla_{\boldsymbol{x}} I(\boldsymbol{x}, \hat{\boldsymbol{s}}) = \kappa I_{\mathrm{b}}(\boldsymbol{x}) - \kappa I(\boldsymbol{x}, \hat{\boldsymbol{s}}) \tag{1}$$

where $\boldsymbol{x}$ is the spatial coordinate, $\hat{\boldsymbol{s}}$ a unit direction vector, κ the absorption coefficient, I the radiative intensity and I_{b} the blackbody intensity (or Planck function). The subscript $\boldsymbol{x}$ on the gradient operator ∇ emphasizes that the gradient is with respect to spatial coordinates only.

In Discrete Ordinate Methods the directional variation of the radiative intensity is expressed by intensities on a set of prescribed directions, known as the ordinates. For each of the n ordinates ($\hat{\boldsymbol{s}}_i, i = 1, \dots, n$), the corresponding intensity ($I_i = I(\boldsymbol{x}, \hat{\boldsymbol{s}}_i)$) is determined by solving the RTE for direction $\hat{\boldsymbol{s}}_i$, i.e.,

$$\hat{\boldsymbol{s}}_i \cdot \nabla I_i(\boldsymbol{x}) = \kappa I_{\mathrm{b}}(\boldsymbol{x}) - \kappa I_i(\boldsymbol{x}) \tag{2}$$

The resulting DOM RTE is a first order partial differential equation and depends on spatial coordinates only. Each ordinate $\hat{\boldsymbol{s}}_i$ has a directional quadrature weight w_i such that a directional integral is converted into a sum over quadratures. In particular,

$$\int d\Omega = \sum_i w_i = 4\pi \tag{3}$$

$$\int I d\Omega = \sum_i I_i w_i = G \tag{4}$$

where G is the incident radiation.

2.2. Finite volume discretization

Eq. (2) may be solved numerically by Finite Volume Methods, i.e., Eq. (2) is integrated over a cell volume before the Gaussian theorem is used to convert differential operations into algebraic operations. For example, consider a cell C with a volume V_c enclosed by n_f faces (Fig. 1). The RTE is integrated over this cell according to

$$\int_{V_c} \hat{\boldsymbol{s}}_i \cdot \nabla I_i(\boldsymbol{x}) dV = \int_{V_c} \kappa I_{\mathrm{b}}(\boldsymbol{x}) dV - \int_{V_c} \kappa I_i(\boldsymbol{x}) dV. \tag{5}$$

For the right-hand side, the cell-center values of radiative intensity (I_{ic}) and properties (κ_c) are defined such that

$$\int_{V_c} \kappa I_{\mathrm{b}}(\boldsymbol{x}) dV - \int_{V_c} \kappa I_i(\boldsymbol{x}) dV = V_c \kappa_c (I_{bc} - I_{ic}) \tag{6}$$