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Review

Linear system approach to the Debye series for electromagnetic scattering by a multi-layer sphere: A tutorial

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ABSTRACT

A general elementary linear system containing two inputs and two outputs is defined, and the behavior of a composite system consisting of a number of elementary systems connected in series is reviewed. In particular, the four proportionality coefficients relating the outputs of the composite system to its inputs have the same formal mathematical structure, independent of the number of elementary systems that are connected together. This composite linear system is then used to model scattering of an electromagnetic plane wave by a singly-coated sphere or a multi-layer sphere. Mirroring the behavior of a general linear system, the partial wave scattering amplitudes and their Debye series representation also have the same formal mathematical structure, independent of the number of layers of the sphere. Lastly, the interpretation of coherent multiple-scattering inside a multi-layer sphere in the frequency-domain is commented on.

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Contents

1. Introduction

For many years, scattering of an electromagnetic plane wave or a transversely focused electromagnetic beam by a coated sphere or a multi-layer sphere has been a problem of both theoretical and experimental interest. The solution

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to the coated sphere scattering problem was first obtained by Aden and Kerker [1], and has since been cast in more streamlined forms [2–5]. Similarly, much progress has been made in developing numerically stable algorithms for computing the quantities of interest for coated sphere scattering [6,7]. The theory of scattering by a multi-layer sphere was developed in [8–14], increasingly stable algorithms for numerical computations were developed in [8–10,15–17], and the extension to scattering by a focused beam was pursued in [18]. In addition, the partial wave scattering and interior amplitudes of a coated sphere [5,19] and a multi-layer sphere [14,20,21] were expressed in terms of the physical processes of diffraction of the partial waves comprising the incident plane wave, external reflection, and transmission following any number of internal reflections. These processes for each partial wave, when added together as an infinite series, are known as the Debye series. Improved algorithms for computing the various the Debye series terms for an absorbing particle were described in [22,23].

In [5,14,21] a number of recurring mathematical patterns were observed for coated sphere and multi-layer sphere scattering in both the partial wave scattering amplitudes and the expressions leading to them. A number of these patterns had previously been encountered by other researchers [4,8,13,24,25]. It was observed that the expressions were formally mathematically identical at every level of assembly of a multi-layer sphere, thus providing an effective iterative approach for computing the partial wave scattering amplitudes. The recurring patterns also imply the action of some basic property or fundamental symmetry of the system that has not been explicitly commented on in the past. The purpose of this tutorial is to clarify the origins of these patterns so as to understand coated-sphere and multi-layer sphere scattering from a more general overarching and fundamental point of view.

The observed patterns, instead of resulting from dynamics particular to electromagnetic scattering, are purely mathematical consequence of the behavior of a general composite linear system. These patterns occur for a large variety of composite systems, requiring only that the quantities under consideration behave linearly, and the elementary constituents that comprise the composite system are connected to each other in series. Electromagnetic scattering by a stratified object satisfies these general requirements since the scalar radiation potential from which the electric and magnetic fields are derived is the solution of a linear differential equation [2,26,27], and the concentric stacking of the different spherical layers satisfies the series connection criterion. The patterns observed here also occur for an incident plane wave propagating through a stack of plane-parallel thin films [28,29]. This leads to the transfer matrix formalism for computing thin film transmission and reflection [30]. They also occur in the adding-doubling method for radiative transfer through a plane-parallel atmosphere [31–33].

The body of this tutorial proceeds as follows. In order to define the notation and set the stage for the more elaborate geometries of Sections 4,5, the partial wave scattering amplitudes for a homogeneous sphere are given in Section 2, and their Debye series representation is derived. As a mathematical interlude, in Section 3 a general linear system containing two inputs and two outputs is described, and the details of the series connection of a number of such systems are recounted. In Section 4 electromagnetic scattering at the interface between two media of differing refractive indices is modeled by an elementary linear system, and scattering by a singly-coated sphere is modeled by the series combination of two such systems. Finally, a multi-layer sphere is modeled in Section 5 by an arbitrary number of elementary systems connected together in series.

The Debye series for scattering by a singly-coated sphere in Section 4.2 and by a multi-layer sphere in Section 5.2 is formally mathematically identical to that for scattering by a homogeneous sphere in Section 2, with the single-interface partial wave transmission and reflection amplitudes now being replaced by multiple-scattering transmission and reflection amplitudes. The linear system point of view described here provides a more straightforward and streamlined derivation of these results than has appeared previously in [5,14,21], and may well be useful in analyzing scattering by a layered object having a more complicated shape. In addition, the form that the proportionality constants of a general composite system assume when they are expressed in terms of the proportionality constants of its constituents motivates the interpretation that coherent multiple-scattering in the frequency-domain, as expressed by the Debye series amplitudes, is mathematically inevitable, rather than resulting from dynamics specific to electromagnetic scattering.

2. Notation and scattering by a homogeneous sphere

Consider a linearly polarized plane wave in an external medium (region 2) of refractive index m_2, having the vacuum wavelength λ, wave number $k=2\pi/\lambda$, electric field strength E_0, traveling in the positive z direction and polarized in the x direction. The time dependence of the plane wave $\exp(-i\omega t)$ will be left implicit for the remainder of this tutorial. The plane wave is incident on and scattered by a homogeneous sphere of radius a (region 1) and refractive index m_1 whose center is at the origin of coordinates. In Lorenz-Mie theory, the scattered fields are expressed as an infinite series of partial wave contributions. The amplitude of the transverse magnetic (TM) scattered fields of partial wave n is standardly denoted by a_n, and the amplitude of the transverse electric (TE) scattered fields is denoted by b_n.

There are four basic partial wave amplitudes in Lorenz-Mie theory in terms of which all the scattering quantities of interest can be expressed,

$$N_n^{12} = \alpha\ \psi_n(m_2ka)\psi'_n(m_1ka) - \beta\psi'_n(m_2ka)\psi_n(m_1ka) \tag{1a}$$

$$D_n^{12} = \alpha\ \chi_n(m_2ka)\psi'_n(m_1ka) - \beta\ \chi'_n(m_2ka)\psi_n(m_1ka) \tag{1b}$$

$$P_n^{12} = \alpha\ \psi_n(m_2ka)\chi'_n(m_1ka) - \beta\ \psi'_n(m_2ka)\chi_n(m_1ka) \tag{1c}$$

$$Q_n^{12} = \alpha\ \chi_n(m_2ka)\chi'_n(m_1ka) - \beta\ \chi'_n(m_2ka)\chi_n(m_1ka) \tag{1d}$$

where $\alpha=m_1$ and $\beta=m_2$ for the TE polarization, $\alpha=m_2$ and $\beta=m_1$ for the TM polarization, and where ψ_n is a Riccati-

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