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Polarized radiative transfer in two-dimensional scattering medium with complex geometries by natural element method

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ABSTRACT

The natural element method (NEM) is extended to solve the polarized radiative transfer problem in a two-dimensional scattering medium with complex geometries, in which the angular space is discretized by the discrete-ordinates approach, and the spatial discretization is conducted by the Galerkin weighted residuals approach. The Laplace interpolation scheme is adopted to obtain the shape functions used in the Galerkin weighted residuals approach. The NEM solution to the vector radiative transfer in a square enclosure filled with a Mie scattering medium is first examined to validate our program. We then study the polarized radiative transfer in two kinds of geometries filled with scattering medium which is equivalent to a suspension of latex spheres in water. Three sizes of spheres are considered. The results for non-dimensional polarized radiative flux along the boundaries and the angular distributions of the Stokes vector at specific positions are presented and discussed. For the complex geometry bounded by the square and circular object, numerical solutions are presented for the cases both with Lambertian (diffuse) reflection and with Fresnel reflection. Some interesting phenomenon are found and analyzed.

1. Introduction

Polarized radiative transfer has found wide applications in the fields of atmospheric radiation [1–3], astronomy, and optical diagnostics of various turbid medium and so on. In these cases, due to the electromagnetic nature of light, the treatment of light reflection, transmission and scattering must include the effects of polarization. This means that the vector radiative transfer theory should be adopted, in which the vector radiative transfer equation (VRTE) for four Stokes parameters needs to be solved. By comparison with the scalar radiative transfer equation, four Stokes parameters need to be solved in the polarized mode. Thus, numerical solution of the VRTE is a complex mathematical procedure. In the last decade, many successful methods have been developed for the problem. A incomplete list of these methods includes the F_N method [4,5], the discrete-ordinates method (DOM) [6–9], the adding and doubling method (ADM) [10], the matrix operator method [11–13], the spherical harmonics method [14], the spherical harmonics discrete ordinates method (SDOM) [15], the successive order of scattering (SOS) method [16,17], the Monte Carlo method (MCM) [18–20] and spectral element method (SEM) [21]. Different methods have different strong and weak points best (as far as accuracy and the efficiency of calculations are concerned) depend on the chosen implementation of general and well-known equations. In addition, significant improvements to the understanding of aspherical particle scattering have been outlined in Brown and Xie [22]. Recently, Brown [23] also made significant

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Nomenclature

$\mathbf{I}$	Stokes vector $\mathbf{I}=(I, Q, U, V)^T$, W/(m^2 μm sr)
$\mathbf{I_0}$	Stokes vector of the incident light
$\mathbf{I}_d$, $\mathbf{I}_c$	Stokes vectors for the diffuse and direct radiation, respectively, W/(m^2 μm sr)
L, H, D	geometrical length of the domain, m
m	complex refractive index
M	total number of discrete directions
n	refractive index
N_θ, N_Φ	numbers of discrete polar angle and azimuthal angle
$\overline{\overline{\mathbf{P}}}$	scattering matrix
$\overline{\overline{\mathbf{R}}}$	reflection matrix
$\mathbf{S}$	source term for radiation, W m^{-2}
$\overline{\overline{\mathbf{T}}}$	transmission matrix for the Fresnel surface
$\overline{\overline{\mathbf{Z}}}$	scattering phase matrix

Greek symbols

ϕ	azimuthal angle
$\overline{\overline{\boldsymbol{\kappa}}}_e$	extinction matrix, m^{-1}
κ_e	extinction coefficient, m^{-1}
κ_s	single scattering coefficient, m^{-1}
μ	direction cosine of the polar angle
θ	polar angle
θ_0, ϕ_0	polar and azimuth angles of the incident light
ρ	reflection coefficient of the Lamberitian surface
$\boldsymbol{\varepsilon}$	emissivity vector
$\boldsymbol{\Omega}$	unit direction vector of radiation

Subscripts

d	diffuse component for Stokes vector
c	collimated component for Stokes vector

Superscripts

T	transpose symbol
m, m'	index for direction
$*$	conjugate transpose

corrections to understanding the geometry and symmetries of full Mueller matrix (full polarization state) backscattering relations for LIDARs in laboratory and planetary settings. An inter-comparison study has been conducted by Kokhanovsky et al. [24] with seven vector radiative transfer codes. Most recently, in order to support model developers and to set standards for polarized radiative transfer modeling, the International Radiation Commission (IRC) has established the working group "International Polarized Radiative Transfer" (IPRT). The first phase of this project was summarized and an inter-comparison of six vector radiative transfer models was readily conducted [25]. The works mentioned above examine the one-dimensional single layer problem. The polarized radiative transfer in multi-layer medium has also attracted the interest of many researchers. An atmosphere–ocean two-layer model of polarized radiative transfer, which is a classical and important problem, has been investigated intensively [26–31].

However, most of the works mentioned above mainly focus on the polarized radiative transfer in the one-dimensional vertical system. Polarized radiative transfer in a multi-dimensional medium is a topic of importance to many areas of study, e.g. biomedical optics [32–36]. For the multi-dimensional polarized problem, the domain concerned has limited geometrical size, and the polarized radiative transfer strongly relies on both the polar and azimuthal angle. The polarized radiative information in all the dimensionalities reveals different features, while the simplified one-dimensional model has limited ability to present the exact information. Thus, the polarized radiative model for multi-dimensional complex geometries needs to be developed, especially for those case involving curved boundaries with different reflection characteristics, e.g., Lambertian (diffuse) reflection or Fresnel reflection.

In this paper, we extend the natural element method (NEM) to solve polarized radiative transfer in two-dimensional scattering medium with complex geometries. The NEM proposed by Braun and Sambridge [37] and Sukumar et al. [38] is a relatively new meshless Galerkin procedure based on the natural neighbor interpolation scheme, which in turn relies on the concepts of Voronoi diagrams and Delaunay triangulation to build Galerkin trial and test functions. Compared to the moving least square technique (MLS) approximation which is widely used in meshless method, some of the most important advantages of natural neighbor interpolants are the properties of interpolation of nodal data, ease of imposing essential boundary conditions, and a well-defined and robust approximation with no user-defined parameter on non-uniform grids. The NEM has advantages of both the finite element method (FEM) and the meshless method and is a promising numerical method for complex computational domains, which has been successfully used to solve the scalar radiative transfer problem in multi-dimensional participating medium [39,40]. However the polarized radiative transfer has not been solved by NEM yet, especially for the multi-dimensional problem with complex geometries involving complicated optical interfaces.

The outline of this paper is as follows. In the following section, the mathematical formulations of the NEM approach for polarized radiative transfer are introduced. Then, the NEM discretization for the VRTE and the solution process for the implementation of NEM is given. In Section 3, the NEM solution for VRTE in a square enclosure filled with Mie scattering medium is first examined to validate our code. Then we study the polarized radiative transfer problem in two kinds of geometries filled with a scattering medium that is equivalent to a suspension of latex spheres in water. Three sizes of spheres are considered. The results for non-dimensional polarized radiative flux along the boundaries and the angular distributions of Stokes vector