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Near-zone evanescent waves generated by weak scattering of light from a spatially deterministic medium

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ABSTRACT

It is commonly known that the far-zone spectrum of a scattered field can be utilized to measure the scattering potential of the medium. However, properties of evanescent fields scattered from the medium with the dielectric susceptibility being a deterministic function, to the best of our knowledge, have not been concerned so far. Assuming the scattering potential of a spatially deterministic medium suffices the Gaussian profile, integrations are derived for the near-zone evanescent field generated by the scattering of light from the medium. It is noticed that the spectral density of the scattered field decays exponentially as either the propagation distance of scattered waves or the effective radius of the scattering potential (ERSP) increases. These results are applicable to the near-field biomedical imaging where the considered tiny particles and molecules solely scatter evanescent waves in near-zone regions.

1. Introduction

Statistical properties of near-zone light waves, which gained considerable research interests in the past few decades, had shown significance in the super-resolution imaging of an inhomogeneous medium [1]. Such work can be done by measuring properties of evanescent surface waves, which can be obtained by using specific scattering experiments. Both theoretical and experimental studies indicated that multiple reflections on interfaces between two different medium emerge when an evanescent surface wave scatters upon a microscopic dielectric sphere [2]. To measure localized photons in the vicinity of surfaces of nano-particles or super-conductive materials, evanescent waves were commonly employed in the near-field scanning optical microscopy (NSOM) [3]. The theorem of imaging formation was introduced by deducing closed-form expressions for near-zone evanescent fields [4]. Based on the analytic singular value decomposition (ASVD) of linear scattering operators, a method was proposed to solve the inverse scattering problem, and analytical expressions were derived to determine the scattering potential of the medium [5]. Furthermore, it was indicated that evanescent waves are capable to reconstruct the scattering potential function of an unknown object [6].

In addition to above investigations, the spectrum [7], spatial correlation [8] and degree of polarization [9,10] of evanescent waves propagating in near-field were concerned, respectively. In general, the scattered field composes of homogeneous and evanescent wave components when a plane wave, which can be either homogeneous or evanescent, scatters upon a spatially random medium [11]. It was exhibited that an incident evanescent wave can

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contribute to generations of both homogeneous and evanescent portions of scattered field [12].

Although [12] held discussions on how the components of homogeneous and evanescent scattered waves are contributed by incident wave components, it did not perform quantitative analyses on properties of scattered evanescent waves in the near field. To address such issue, we performed numerical simulations of the spectral shifts of evanescent waves generated by the weak scattering of light upon a spatially deterministic medium [13]. Moreover, we also investigated the effects of the anisotropy of the scattering potential of the medium on distributions of spectral densities of evanescent waves in near-zone scattered field [14]. To date, however, no existing literature has concerned properties of spectral densities of scattered evanescent waves, which may strongly depend on parameters of incident beams and the medium. As a contribution study, in this paper we attempt to derive expressions for the near-field evanescent waves scattered from a spatially deterministic medium, while the first-order Born approximation is applied to obtain the primary results. Also, close attentions will be paid to spatial distributions of spectral densities of evanescent waves. By assuming the scattering potential of the medium suffices the Gaussian profile, the dependences of spectral densities of scattered fields on the propagation distance of scattered waves, ERSP and radial length of the medium are displayed by numerical simulations, respectively.

2. Evanescent waves generated by scattering of light from a spatially deterministic medium

To begin with, let us recall the scattering theory of a monochromatic plane wave from a spatially deterministic medium. The electric field of the incident plane wave is given by:

$$U^{(i)}\left(\vec{\boldsymbol{r}},\omega_0\right)=\exp\left(ik_0\vec{\boldsymbol{s}}_{\boldsymbol{0}}\cdot\vec{\boldsymbol{r}}\right), \tag{1}$$

where ω_0 is the frequency, $k_0=\omega_0/c$ denotes the wave number, $\vec{s}_{\boldsymbol{0}}$ stands for the unit vector which represents the propagation direction of the plane wave, the medium occupies a finite volume D. We consider the scattering of light from the medium is so weak, therefore, the total electric field can be expressed as the summation of the incident and scattered fields:

$$U^{(t)}\left(\vec{\boldsymbol{r}},t\right)=\left[U^{(i)}\left(\vec{\boldsymbol{r}},\omega_0\right)+U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)\right]\exp(-i\omega_0 t). \tag{2}$$

In Eq. (2), $U^{(s)}(\vec{\boldsymbol{r}},\omega_0)$ must suffice the elementary wave equation [11,12,15]:

$$\left(\nabla^2+k_0^2\right)U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)=-F\left(\vec{\boldsymbol{r}},\omega_0\right)\left[\exp\left(ik_0\vec{\boldsymbol{s}}_{\boldsymbol{0}}\cdot\vec{\boldsymbol{r}}\right)+U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)\right], \tag{3}$$

where $\nabla^2=\partial^2/\partial x^2+\partial^2/\partial y^2+\partial^2/\partial^2 z$ is the Laplacian operator in the 3D Cartesian coordinate, $F\left(\vec{\boldsymbol{r}},\omega_0\right)$ denotes the scattering potential of the medium. Because the scattered field is sufficiently weak compared with that of the incident field, i.e. $|U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)|<<\left|U^{(i)}\left(\vec{\boldsymbol{r}},\omega_0\right)\right|$, the first-order Born approximation can be employed to solve Eq. (3). As a result, Eq. (3) results in the following integration:

$$U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)=\int_D\frac{\exp\left(ik_0\left|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'\right|\right)}{\left|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'\right|}F\left(\vec{\boldsymbol{r}}',\omega_0\right)\exp\left(ik_0\vec{\boldsymbol{s}}_{\boldsymbol{0}}\cdot\vec{\boldsymbol{r}}'\right)d^3r'. \tag{4}$$

Alternatively, Eq. (4) can be also rewritten as the angular spectrum representation of plane waves. For such purpose, the outgoing spherical wave function in Eq. (4) must be substituted by the Weyl representation [11,12,15]:

$$\frac{\exp\left(ik_0\left|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'\right|\right)}{\left|\vec{\boldsymbol{r}}-\vec{\boldsymbol{r}}'\right|}=\frac{ik_0}{2\pi}\int_D\frac{1}{s_z}\exp\left\{ik_0\left[\vec{\boldsymbol{s}}_\perp\cdot\left(\vec{\boldsymbol{\rho}}-\vec{\boldsymbol{\rho}}'\right)+s_z|z-z'|\right]\right\}d^2s_\perp, \tag{5}$$

where

$$s_z=\begin{cases}\left(1-s_\perp^2\right)^{1/2}, & s_\perp^2\le 1\\ i\left(s_\perp^2-1\right)^{1/2}, & s_\perp^2>1\end{cases}, \tag{6}$$

and $\vec{\boldsymbol{s}}_\perp=(s_x,s_y,0)$, $\vec{\boldsymbol{r}}=\left(\vec{\boldsymbol{\rho}},z\right)$, $\vec{\boldsymbol{r}}'=\left(\vec{\boldsymbol{\rho}}',z'\right)$. The subscript "D" in Eq. (5) means the integration is over the entire scatterer volume. By substituting Eq. (5) into (4), the scattered field can be given by:

$$U^{(s)}\left(\vec{\boldsymbol{r}},\omega_0\right)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}a^{(s)}\left(\vec{\boldsymbol{s}}_\perp,\omega_0\right)\exp\left[ik_0\left(\vec{\boldsymbol{s}}_\perp\cdot\vec{\boldsymbol{\rho}}+s_z z\right)\right]d^2s_\perp, \tag{7}$$

where $a^{(s)}\left(\vec{\boldsymbol{s}}_\perp,\omega_0\right)$ denotes the spectral amplitude function, which is shown as the following integral form:

$$a^{(s)}\left(\vec{\boldsymbol{s}}_\perp,\omega_0\right)=\frac{ik_0}{8\pi^2 s_z}\int_D F\left(\vec{\boldsymbol{r}}',\omega_0\right)\exp\left\{-ik_0\left[\vec{\boldsymbol{s}}_\perp\cdot\vec{\boldsymbol{\rho}}'+(s_z-1)z'\right]\right\}d^3r'. \tag{8}$$

Thereinafter, we focus on properties of near-zone evanescent waves of the scattered field. Accordingly, s_z must satisfy the bottom option of Eq. (6). As a result, the spectral amplitude function is exhibited as the following form (see Eq. (13) of [11]):

$$a_e^{(s)}\left(\vec{\boldsymbol{s}}_\perp,\omega_0\right)=\frac{\pi k_0}{|s_z|}\tilde{F}\left[k_0 s_x,k_0 s_y,k_0(i|s_z|-1)\right], \tag{9}$$

where $\tilde{F}\left(\vec{\mathbf{K}}\right)$ is the 3D Fourier transform of the scattering potential:

$$\tilde{F}\left(\vec{\boldsymbol{K}}\right)=\frac{1}{(2\pi)^3}\int_D F\left(\vec{\boldsymbol{r}}',\omega_0\right)\exp\left(-i\vec{\boldsymbol{K}}\cdot\vec{\boldsymbol{r}}'\right)d^3r'. \tag{10}$$

Eqs. (9) and (10) show that the spectral amplitude function is proportional to the Fourier transform of the scattering potential of the medium. For simplicity, we assume that the scattering potential satisfies the Gaussian profile [16]:

$$F\left(\vec{\boldsymbol{r}}',\omega_0\right)=\frac{A}{\left(2\pi\delta^2\right)^{3/2}}\exp\left[-\vec{\boldsymbol{r}}'^2/2\delta^2\right], \tag{11}$$