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Scattering of light by large bubbles: Coupling of geometrical and physical optics approximations

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ABSTRACT

This paper analyzes various phenomena in modeling the light-scattering properties of large spherical bubbles in the context of geometrical and physical optics approximations. Among these phenomena are interference occurring between higher-order rays, the Goos–Hänchen shift, the tunneling phase and the weak caustic associated with the critical angle. When the phenomena are appropriately taken into account, they allow retrieval of most features of the scattering diagrams predicted by the Lorenz–Mie theory, offering new possibilities for the optical characterization of bubbly flows.

1. Introduction

The electromagnetic-light-scattering community recently celebrated the one-hundredth anniversary of the publication in 1908 of what is now referred to as the Mie or Lorenz–Mie theory (LMT) [1]. The latter solves in an exact manner, with a separation variable method, the problem of absorption and scattering by a spherical, homogeneous, isotropic and non-magnetic particle (Mie scatterer) when illuminated by an incident electromagnetic plane wave. Since then, the LMT has been generalized to account for shaped and pulsed illuminating beams [2–6] as well as highly symmetrical particles (e.g., multilayered or chiral spheres, spheroids and cylinders) [6–9]. In a concomitant way, various other electromagnetic methods (e.g., time-domain or discrete multipole decomposition and null-field) [10–13] have been developed to address particles of nearly arbitrary shape. However, because of numerical solution complexity and computational limitations, all the latter exact methods are currently limited to particles with a maximum size parameter of typically 10–600 (e.g. [13]).This upper limit is far below the need for direct and inverse optical characterization of two-phase or bubbly flows [14–19], in which non-spherical particles are frequently encountered with diameters of a few hundred microns to a few millimeters. This limit does not apply in the classical LMT, even though the accurate numerical calculation of the optical scattering properties of millimeter-scale particles becomes difficult and time consuming (e.g., up to a few minutes on a desktop computer). This explains why the development of alternative approaches allowing fast (i.e., tens of milliseconds) and accurate calculations (at least asymptotically) are still needed. The geometrical optics approximation (GOA) is one of the preferred approaches for large particles because it provides physical insight, is computationally efficient and is straightforward to apply when extended to complex-shaped

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particles (e.g., homogeneous and coated spheres [20–24], spheroids [25–27], cylinders [28,29], crystals [10,29] and particles with a rough surface [30]).

Because GOA cannot account for pure wave effects that are crucial for optical particle characterization methods [31–33] or for an energy balance (e.g., forward diffraction contributes up to 50% of the scattering cross section of large particles) [34], it is desirable to couple GOAs with *ad hoc* physical optics approximations (POAs). Among the POAs there are, for instance, classical analytical methods like Fraunhofer approximation and Airy's theory of the rainbow (e.g., [35,36]) as well as hybrid methods (e.g., [37,38]). Indeed, the implementation and coupling of various GOAs and POAs into a unified geometrical and physical optics approximation (GPOA) is not trivial, even in the case of simple Mie scatterers. This is true in particular for bubbles and notably in the vicinity of the critical scattering angle. Fig. 1 shows the calculation with LMT of the near-field power inside and outside an air bubble in water with relative index $m = 0.75$ and radius $R = 50$ μm for a parallel polarized plane wave with wavelength in air of $\lambda_0 = 0.6328$ μm. The latter figure clearly illustrates the underlying complexity of the problem (e.g. sharp intensity gradients and interferences phenomena connected to the total reflection and the associated evanescence wave).

To model the scattering of bubbles in the framework of GOA, several important points need to be considered: (i) the Fresnel coefficients associated with the critical incident angle $i_c = \sin^{-1}(m)$ are only once differentiable, giving rise to a weak caustic; (ii) a tunneling effect exists for the critical rays; and (iii) strong interferences exist between higher-order rays. Marston and co-workers [39–41] have developed, using the framework of scalar diffraction theory, a physical optics approximation (denoted by M-POA to avoid any confusion) to account for points (i) and (ii). Fiedler-Ferrari et al. [42] had the same goal

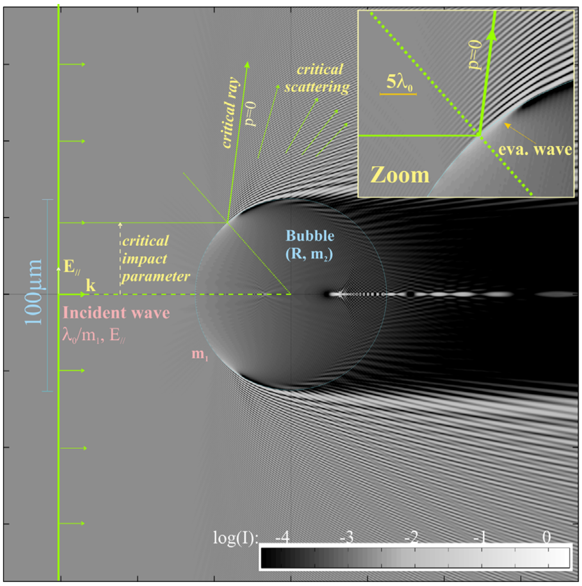

Fig. 1. LMT calculation of the near-field power inside and outside an air bubble in water ($R = 50$ μm, $m = 0.75$, parallel pol., $\lambda_0 = 0.6328$ μm).

when they developed a zero-order approximation of the near-critical-angle scattering (derived from the complex angular momentum theory [36]). Despite the great scientific value of these two semiclassical approximations, they do not provide a full description of the fine structures of the scattering diagrams, and their validity is limited to a narrow angular range. Therefore, although recently GOA and POA have been proposed to account for bubbles or droplets with more complex shapes [43,44,25], the case of spherical bubbles remains topical and LMT remains a unique tool for validation purposes.

The present paper compares and improves several approaches to account for points (i)–(iii), with focus being given to the near-critical-angle scattering region. The remainder of the paper proceeds as follows. In Section 2 the founding principles of Van de Hulst's GOA [24] are reviewed and numerical results are presented for the purposes of comparison and discussion. In Section 3 two ways to account for the tunneling phase and the Goos–Hänchen effect are proposed. Section 4 details various considerations to extend and implement Marston's approximation in the previously introduced models. Finally, Section 5 presents the overall conclusions with perspectives.

2. Van de Hulst model: basic formalism (GPOA-1)

We consider a spherical bubble of radius R centered in the laboratory frame ($Oxyz$). It is illuminated by a parallel (subscript $\chi = \parallel$) or perpendicular ($\chi = \perp$) polarized and harmonic plane wave propagating along the y-axis with wavelength in air equal to λ_0; see Fig. 2. For this particular wavelength, the bubble medium and surrounding medium refractive indices are equal to m_1 and m_2 respectively, with $m = m_2/m_1 < 1$ denoting the relative refractive index. This study is restricted to bubbles with a large size parameter (i.e., $\alpha = 2\pi R m_1/\lambda_0 \gg 1$).

In Van de Hulst's GOA [24], which is expected to be valid for both bubbles ($m < 1$) and refracting particles ($m > 1$), the incident plane wave is decomposed into rays characterized by a scattering order $p = 0, 1, \ldots, +\infty$ and by two incident angles, i_1 and i_2 (see Figs. 2 and 3). The corresponding complementary angles τ_1 and τ_2 are related by the Snell–Descartes law:

$$\cos(\tau_1) = m \cos(\tau_2) \qquad (1)$$

The ray of order p emerges (after $p-1$ internal reflections) in the direction defined by the scattering angle θ_p [24] with

$$\theta_p = 2\tau_1 - 2p\tau_2, \quad \text{for } \tau_1 \in [-\pi/2, \pi/2] \qquad (2)$$

Because of the symmetry of the problem, and because it is more convenient to address rays emerging exclusively in the range $\theta_p \in [0, \pi]$, Eq. (2) can be completed as follows:

$$\theta_p = 2\kappa\pi + q[2\tau_1 - 2p\tau_2], \quad \text{for } \tau_1 \in [0, \pi/2] \qquad (3)$$

where κ is an integer allowing compensation for the number of rotations (which can be important for higher-order rays) and $q = \pm 1$ allows restricting the scattering domain to $\theta_p \in [0, \pi]$. Total external reflection occurs for