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Conditions for invariant spectrum of light generated by scattering of partially coherent wave from quasi-homogeneous medium

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ABSTRACT

Within the first-order Born approximation, the spectrum of light generated by the scattering of a partially coherent wave from a quasi-homogeneous (QH) medium is derived. In particular, the partially coherent incident wave is produced by Young's pinholes. It is shown that the spectrum of the scattered field is identical to the spectrum of incident plane waves if the Fourier transform of the normalized correlation coefficient (NCC) of the scattering potential satisfies a certain scaling law. The scaling law is valid when the medium size is sufficiently small compared with the space between Young' pinholes. Furthermore, comparisons are made between our conditions with the previous results.

1. Introduction

Quasi-homogeneous (QH) sources have received much attention in the past few decades and found their ways into a number of applications, like high-resolution imaging, free space optical communications (FSOC) and measurements of beam qualities [1–3]. It was shown that intensities of light radiated from a QH source vary slowly with positions and are invariant over long distances. Polarization properties are studied for the beam generated by a partially polarized QH source propagating through polarization gratings [4]. A modified scaling law was obtained for the invariance of the spectrum of light generated by a planar, secondary QH source of any state of coherence [5]. Subsequently, it was exhibited that the modified scaling law can be generalized to the case where the propagation of light through a turbulent medium was considered [6]. Based on the unified theory of coherence and polarization of electromagnetic beams, the cross-spectral density matrix of beams radiated from an electromagnetic QH source was derived [7,8]. In addition, changes of instantaneous Stokes parameters of an electromagnetic QH beam were found to have strong dependences on two-point correlations between field components [9].

Besides, analytical expressions for the coherence and polarization of light radiated from 3D, primary and QH sources were obtained [10,11]. Conditions for the invariant polarization of a beam were derived [12]. The generalized van Cittert-Zernike theorem for the cross-spectral density matrix of an electromagnetic QH source was introduced [13].

In addition to aforementioned researches, extensive studies on the scattering of light waves from a spatially QH medium were also performed. For example, the diffraction tomography which is helpful to determine a QH random

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medium was investigated for a scattered field [14]. The inverse scattering problem for a QH medium was solved by utilizing a plane wave and a pulse wave upon scattering [15,16], respectively. Reciprocal relations between the far-zone spectral densities and the spectral degree of coherence (SDOC) of a scattered field were obtained [17–19]. Correlations between intensity fluctuations of light scattered from a QH medium were investigated [20–22]. Furthermore, the scattering of light from a QH anisotropic medium was addressed in a sequence of reports [23–26]. Effects of anisotropic properties of a medium on the spectrum, SDOC and the spectral degree of polarization (SDOP) of a scattered field were quantitatively analyzed [27–30].

All results described above were performed by assuming specified functions of the SDOC of incident waves. However, incident waves that scatter upon a medium can be an arbitrarily partially coherent beam in practice. Therefore, a model which describes the scattering of partially coherent waves from a spatially random medium is required. To this end, here we intend to obtain the spectrum of the scattered field by considering a partially coherent light on incidence. Also, we derive a sufficient condition to ensure the invariant spectrum of the scattered field. Our findings might be beneficial to determinations of a spatially random medium and optimizations of beam qualities in FSOC applications.

2. Condition for the invariance of spectrum of light generated by scattering of a partially coherent wave from a QH medium

Assume that a plane wave transmits through an opaque screen A which occupies pinholes $Q\left(\vec{\rho}_{1\perp}\right)$ and $Q\left(\vec{\rho}_{2\perp}\right)$, as shown in Fig. 1. $\vec{\rho}_{1\perp}$ and $\vec{\rho}_{2\perp}$ represent the projective vectors of $\vec{\rho}_1$ and $\vec{\rho}_2$ onto the screen A, respectively. φ_1 and φ_2 are the azimuthal angles between $\vec{\rho}_1, \vec{\rho}_2$ and the unit vector $\vec{s}_0$, respectively. d is the space between two pinholes. The electric field of the incident plane wave is given by:

$$U^{(i)}\left(\vec{\rho}_j, \omega\right) = a(\omega)\exp\left(-ik\vec{s}_0 \cdot \vec{\rho}_j\right), \ (j=1,2), \tag{1}$$

where $a(\omega)$ is the spectral amplitude which depends on the frequency. $k=2\pi/\lambda$ is the wave number, λ is the wavelength of the light. $\vec{s}_0$ denotes the unit vector which represents the propagation direction. The superscript i stands for the incident field. $P\left(\vec{r}_1'\right)$ and $P\left(\vec{r}_2'\right)$ are the points where the transmitted wave interacts with a spatially QH medium. $\vec{r}_1$ and $\vec{r}_2$ are the position vectors in the scattered field, $\vec{s}_1$ and $\vec{s}_2$ are the unit vectors of $\vec{r}_1$ and $\vec{r}_2$, respectively. θ_1 and θ_2 are the scattering angles. By introducing the ensemble average of the electric field $\left\langle U^{(t)}\left(\vec{r}_j', \omega\right)\right\rangle$ at $P\left(\vec{r}_j'\right)$, the connection between $\left\langle U^{(i)}\left(\vec{\rho}_j', \omega\right)\right\rangle$ and $\left\langle U^{(t)}\left(\vec{r}_j', \omega\right)\right\rangle$ can be obtained as [31–33]:

$$\begin{pmatrix} U^{(t)}\left(\vec{r}_1', \omega\right) \\ U^{(t)}\left(\vec{r}_2', \omega\right) \end{pmatrix} = \begin{pmatrix} -\frac{ik\exp(ikR_{11})}{2\pi R_{11}}dS & -\frac{ik\exp(ikR_{12})}{2\pi R_{12}}dS \\ -\frac{ik\exp(ikR_{21})}{2\pi R_{21}}dS & -\frac{ik\exp(ikR_{22})}{2\pi R_{22}}dS \end{pmatrix} \cdot \begin{pmatrix} U^{(i)}\left(\vec{\rho}_1, \omega\right) \\ U^{(i)}\left(\vec{\rho}_2, \omega\right) \end{pmatrix}, \tag{2}$$

where $R_{\alpha\beta}$ $(\alpha, \beta=1,2)$ denotes the distance between $P\left(\vec{r}_\alpha'\right)$ and $Q\left(\vec{\rho}_\beta\right)$. dS represents the pinhole area. Recalling the expression for the cross-spectral density function (CSDF) of a statistical, stationary field [31,34]:

$$W\left(\vec{r}_1', \vec{r}_2', \omega\right) = \left\langle U^*\left(\vec{r}_1', \omega\right) U\left(\vec{r}_2', \omega\right)\right\rangle, \tag{3}$$

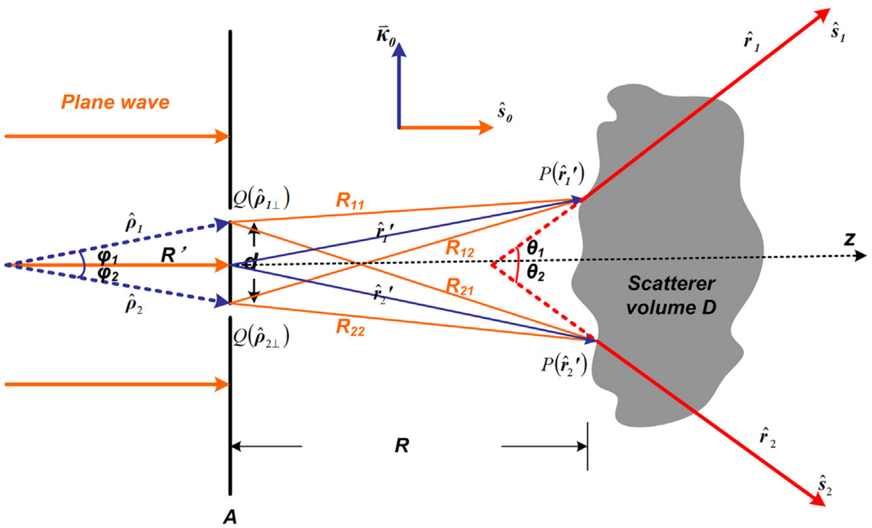

Fig. 1. Notation of the scattearing of a partially coherent wave from Young's pinholes.