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Anomalous change of Airy disk with changing size of spherical particles

Linchao Pan [a,b], Fugen Zhang [a,b], Rui Meng [a,b], Jie Xu [a,b], Chenze Zuo [a,b], Baozhen Ge [a,b,*]

[a] School of Precision Instrument and Opto-Electronics Engineering, Tianjin University, Tianjin 300072, China
[b] Key Laboratory of Opto-Electronics Information Technology of Ministry of Education, Tianjin 300072, China

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A B S T R A C T

Use of laser diffraction is considered as a method of reliable principle and mature technique in measurements of particle size distributions. It is generally accepted that for a certain relative refractive index, the size of the scattering pattern (also called Airy disk) of spherical particles monotonically decreases with increasing particle size. This fine structure forms the foundation of the laser diffraction method. Here we show that the Airy disk size of non-absorbing spherical particles becomes larger with increasing particle size in certain size ranges. To learn more about this anomalous change of Airy disk (ACAD), we present images of Airy disk and curves of Airy disk size versus particle size for spherical particles of different relative refractive indices by using Mie theory. These figures reveal that ACAD occurs periodically for non-absorbing particles and will disappear when the absorbing efficiency is higher than certain value. Then by using geometrical optics (GO) approximation, we derive the analytical formulae for the bounds of the size ranges where ACAD occurs. From the formulae, we obtain laws of ACAD as follows: (1) for non-absorbing particles, ACAD occurs periodically, and when the particle size tends to infinity, the period tends to a certain value. As the relative refractive index increases, (2) the particle size ranges where ACAD occurs shift to smaller values, (3) the period of ACAD becomes smaller, and (4) the width of the size ranges where ACAD occurs becomes narrower. In addition, we can predict from the formulae that ACAD also exists for particles whose relative refractive index is smaller than 1.

1. Introduction

The method of characterizing particles by measuring the light that they scatter is proposed in the 1960s and has been used for nearly 50 years. Instruments based on this method (hereinafter referred to as the laser diffraction instruments) have been widely used in scientific research and production of powders, liquid sprays, and suspensions. The laser diffraction instrument is considered as an equipment of reliable principle and mature technique. There are two versions of ISO standards for this instrument [1,2]. The physical foundation of the instrument is Mie theory, which based on Maxwell's equations. Mie theory is a rigorous solution for the light scattered by a spherical, homogeneous, isotropic, and non-magnetic particle in a non-absorbing medium [3]. If the particle size is much larger than the wavelength of light, Fraunhofer diffraction theory can also give a good description of the light scattered in the near-forward directions. Van de Hulst [4] reported that for particles that satisfy the conditions of anomalous diffraction ($\alpha \gg 1$ and $|m-1| \ll 1$, where $\alpha = \pi D/\lambda$ is the size parameter, D is the diameter of the particle, λ is the

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[*] Corresponding author at: School of Precision Instrument and Opto-Electronics Engineering, Tianjin University, Tianjin 300072, China.
E-mail address: gebz@tju.edu.cn (B. Ge).

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wavelength of light in the dispersion medium, $m = m_1/m_2$ is the refractive index of the particle relative to the dispersion medium), Fraunhofer diffraction cannot give a good approximation of the scattered light of spherical particles in the near-forward directions. Following van de Hulst's work, Han et al. [5] investigated the disagreement of diffraction theory and Mie theory in the near-forward directions for different refractive indices. Shen et al. [6] obtained different approximations of the diffraction light and compared with the rigorous calculations obtained by Mie theory. Lock et al. [7] and Laven [8] investigated the scattering of sunlight by using Mie theory and diffraction theory. In this paper, we focus on the change of Airy disk size with changing size of spherical particles. In Fraunhofer diffraction theory, the scattering particle is regarded as a totally opaque disk of the same diameter, and its far-field scattering pattern is composed of a central bright spot gradually decaying from inside out and a series of concentric rings. Such scattering pattern is called the Airy disk [3]. The Airy disk size is described by the angular radius θ_A at which the first minimum of the scattering pattern intensity occurs. In fact, as long as the diameter of the spherical particle is close to or larger than the wavelength of light, the scattering pattern has signatures similar to those of a totally opaque disk of the same diameter. Therefore, the scattering pattern of this type can also be called Airy disk and its size can be described by the size of the Airy disk. It is generally accepted that the Airy disk size monotonically decreases with increasing particle size. This fine structure forms the foundation of the laser diffraction method [3].

In fact, the prevailing understanding of the change of Airy disk size is not entirely correct. Van de Hulst [4] reported that for particles that satisfy the conditions of anomalous diffraction and water drops in air ($m_1 = 1.33$, $m_2 = 1$), the change of Airy disk size with changing particle size is not monotonic. In certain size ranges, there are oscillations. That is, sometimes the Airy disk size becomes larger with increasing particle size. This anomalous change of Airy disk (ACAD) can be interpreted by anomalous diffraction theory: the interference between the diffracted light and the refracted light causes the scattering pattern of a non-absorbing particle to have some different signatures compared with a totally opaque disk of the same diameter. Our research group has long been engaged in the study of laser diffraction method, and we have repeatedly found that for particles in certain size ranges of certain relative refractive indices (for example, polystyrene microspheres ($D = 3$ μm) dispersing in water), the measured particle size distribution is incorrect when using a laser diffraction instrument [9]. The reason is clearly not the original signal error. We believe ACAD is the real reason that leads to such measurement error.

In this paper, we conduct a further and more comprehensive analysis of ACAD and obtain some laws. First, we present images of Airy disk and curves of Airy disk size versus particle size for spherical particles of different relative refractive indices by using Mie theory and verify the general existence of ACAD for non-absorbing particles. Then by using geometrical optics (GO) approximation, we derive analytical formulae for the bounds of the size ranges where ACAD occurs (ACAD ranges for simplicity). According to the formulae, we obtain some laws about ACAD and discuss the applicability of the formulae when the relative refractive index is very high and less than 1.

2. Description

To illustrate ACAD, we present images of scattering patterns of spherical particles by using Mie theory. Here, we suppose that the wavelength of the incident light $\lambda = 0.6328$ μm and the refractive index of the dispersion medium $m_2 = 1.33$. Fig. 1(a1)–(a4) illustrate the scattering patterns of spherical particles ($m = 1.2$) illuminated by unpolarized light for four size parameters: $\alpha = 19$, 23, 35, and 40 (which correspond to particle diameters of 2.88 μm, 3.48 μm, 5.30 μm, and 6.06 μm). Although the figures from left to right show a gradually increasing diameter, the Airy disk size is not

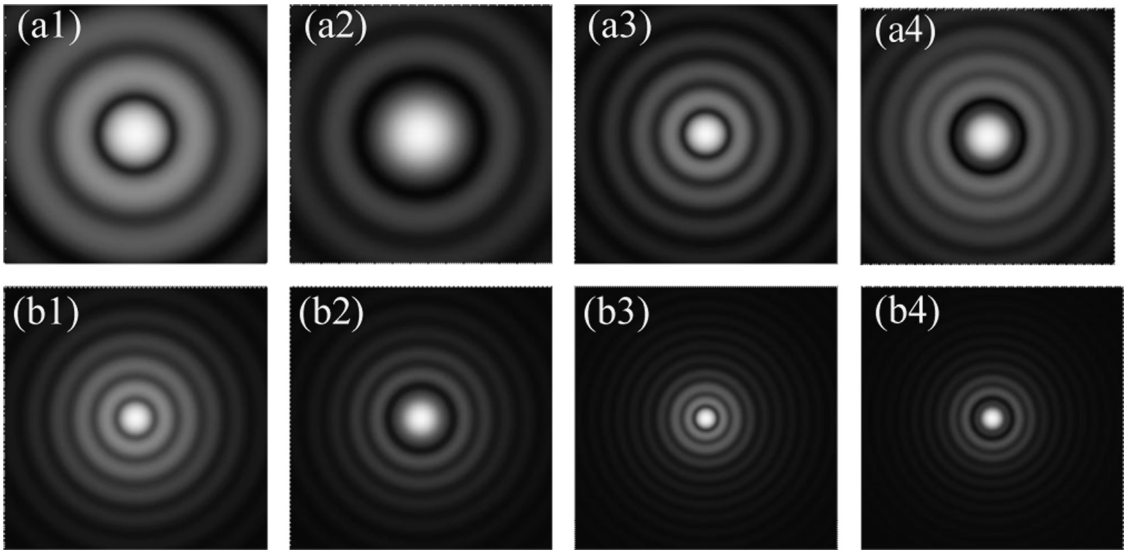

Fig. 1. Mie scattering patterns for spherical particles, (a1)–(a4) are obtained when $m = 1.2$, (b1)–(b4) are obtained when $m = 1.1$. (a1) $\alpha = 19$, $\theta_A = 8.09°$ (a2) $\alpha = 23$, $\theta_A = 13.06°$ (a3) $\alpha = 35$, $\theta_A = 5.08°$ (a4) $\alpha = 40$, $\theta_A = 7.90°$ (b1) $\alpha = 39$, $\theta_A = 4.24°$ (b2) $\alpha = 45$, $\theta_A = 7.02°$ (b3) $\alpha = 72$, $\theta_A = 2.61°$ and (b4) $\alpha = 78$, $\theta_A = 4.35°$.