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Multi-scale methods for the solution of the radiative transfer equation

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A B S T R A C T

Various methods have been developed and tested over the years to solve the radiative transfer equation (RTE) with different results and trade-offs. Although the RTE is extensively used, the approximate diffusion equation is sometimes preferred, particularly in optically thick media, due to the lower computational requirements. Recently, multi-scale models, namely the domain decomposition methods, the micro–macro model and the hybrid transport–diffusion model, have been proposed as an alternative to the RTE. In domain decomposition methods, the domain is split into two subdomains, namely a mesoscopic subdomain where the RTE is solved and a macroscopic subdomain where the diffusion equation is solved. In the micro–macro and hybrid transport–diffusion models, the radiation intensity is decomposed into a macroscopic component and a mesoscopic one. In both cases, the aim is to reduce the computational requirements, while maintaining the accuracy, or to improve the accuracy for similar computational requirements. In this paper, these multi-scale methods are described, and the application of the micro–macro and hybrid transport–diffusion models to three-dimensional transient problems is reported. It is shown that when the diffusion approximation is accurate, but not over the entire domain, the multi-scale methods may improve the solution accuracy in comparison with the solution of the RTE. The order of accuracy of the numerical schemes and the radiative properties of the medium play a key role in the performance of the multi-scale methods.

1. Introduction

Many problems in mathematics, biology, physics, chemistry and engineering encompass different spatial and/or time scales. The mathematical modeling of such problems requires a multi-scale approach. This may be defined as a "general framework for formulation and design of methods which model a system's behavior governed by a hierarchy of scales, both spatial and temporal, and their interactions, and provide a seamlessly coupled platform through which the interacting scales mutually exchange their information" [1]. The range of relevant length and time scales in materials science, for example, includes the electronic, atomic, microscopic, mesoscopic and macroscopic, or continuum scales [2]. An efficient numerical solution of a multi-scale problem requires that appropriate numerical methods are used for every relevant scale, and coupled to allow effective data exchange between different scales.

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In some multi-scale problems there is no clear scale separation, as in the case of turbulent fluid flow and in radiative transfer. The modeling and numerical simulation of radiative transfer phenomena has been a very active field of research during the last decades. Indeed, radiative transfer underlies numerous technological applications (e.g., combustion, optical tomography, solar energy), as well as more fundamental research. Because of the multi-scale character of most radiative transfer phenomena, and since analytical solutions are available only in a few simplified cases, the numerical simulation of radiative transfer phenomena is still a challenging task nowadays. According to the physical context, radiative transfer phenomena can be modeled by means of two classes of mathematical models: the radiative transfer equation (RTE) and the diffusion equation (DE) (see [3]).

At the top of the hierarchy of models, the RTE appears as the reference from a modeling point of view. The RTE accurately describes radiation transport in media under local thermal equilibrium, yielding the time evolution of the radiation intensity, which depends on 6 dimensions (space, direction and wavelength) plus time. However, the numerical solution of the RTE may be computationally expensive, particularly when the optical thickness of the medium is high. The discrete ordinates method (DOM), the finite volume method (FVM) and the Monte Carlo (MC) method become inefficient in the so-called diffusive regime. Indeed, the numerical parameters for the DOM or FVM methods, such as the grid size and time step, must satisfy severe constraints for stability reasons. As far as the Monte Carlo method is concerned, the computational cost may be prohibitive due to the high number of scattering events, and the numerical convergence becomes slower.

At the bottom of the hierarchy, the DE describes radiative transfer at the macroscopic scale, yielding the incident radiation, which depends on time, the three spatial dimensions and wavelength. The DE is based on the assumption that the radiation intensity is nearly isotropic. Obviously, when this assumption is not valid, which is the case when the medium or part of the medium is not optically thick, or when boundary conditions or radiative sources have a strong influence on radiative transfer, the results are inaccurate. Moreover, finding accurate boundary conditions for the DE may be a complicated problem.

The required computational time for the solution of the RTE is much larger than that for the solution of the DE for realistic multi-dimensional simulations. Hence, several computational methods have been developed that combine the RTE and the DE, aiming at the improvement of the computational efficiency of radiative transfer simulations without compromising the solution accuracy [4]. These methods are an interesting option in problems where diffusive and kinetic regimes coexist, e.g., media with a low-scattering region and a strongly scattering region, or in problems where multiple spatial and temporal scales are present. This multi-scale character makes the development of numerical models and the associated simulations a real challenge.

In this work, our goal is to report recent advances on multi-scale models for radiative transfer, namely a domain decomposition strategy [5], a micro–macro model [6] and a hybrid transport–diffusion model [7], are presented. These approaches involve an interesting trade-off between accuracy and computational requirements and offer a good alternative to the use of a full DE or RTE.

One possible strategy to deal with multi-scale models is to couple the DE and the RTE through a spatial domain decomposition approach (DD), where the domain is decomposed into a mesoscopic subdomain, in which the RTE is solved, and a diffusive one, in which the DE is solved. Several variants of the DD method have been reported, depending on the numerical methods used to solve the RTE and the DE, and on how the coupling between the two subdomains is implemented [4,8–11]. Most of these works have been developed for light propagation in tissues for biomedical applications. The treatment of the interface between the macroscopic and the microscopic zones represents a critical issue. In fact, the boundary conditions of the DE must be consistent with the boundary conditions of the RTE, which is not an easy task. Hence, a buffer zone between the kinetic and diffusive subdomains was introduced to overcome this issue [5,12], which avoids the need to define boundary conditions for the DE.

Another strategy is to apply a multi-scale model in the entire domain. A first model presented in this work is the micro–macro (MM) model, which was originally developed in other research fields [13–15], and recently applied to the radiative transfer equation [16]. It is based on the decomposition of the dependent variable of the governing equation into a macroscopic component and a mesoscopic one. The micro–macro model satisfied by the incident radiation, G, plus a correction, ε, is equivalent to the RTE. The solution of the RTE is recovered by simply adding the contributions of the macro (G) and kinetic (ε) components. The resulting model involves a two-way coupled system, the numerical approximation of which needs artificial boundary conditions for the macroscopic unknown G. Moreover, due to the two-way coupling, a Monte Carlo method cannot be directly applied to the system. However, it was shown that some improvements are observed when grid based methods are employed, compared to the numerical solution of the full RTE, especially when the system is close to the diffusive regime [16].

The last model described in this paper, referred to as hybrid transport–diffusion (HTD) model, relies also on a decomposition of the radiation intensity [7]. In fact, expressing the radiation intensity as the sum of G^{lim}, where G^{lim} is the solution of the DE, plus a correction ε, one can write a one-way coupled model satisfied by G^{lim} and ε. An approximate solution of the RTE is calculated by adding the macroscopic and the kinetic components, as in the MM model. However, in contrast to the MM model, a Monte Carlo method can be easily applied to the kinetic equation. Indeed, since the DE does not depend on the kinetic component ε, it can be solved independently on the whole time-space interval under consideration. Then, a Monte Carlo method can be easily used since the kinetic part of the HTD system only differs from the RTE by the presence of a source term depending on G^{lim}. Hence, the linear structure is preserved, which is a real advantage when using a Monte Carlo method. Moreover, since