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Radiative heat transfer in strongly forward scattering media using the discrete ordinates method

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ABSTRACT

The discrete ordinates method (DOM) is widely used to solve the radiative transfer equation, often yielding satisfactory results. However, in the presence of strongly forward scattering media, this method does not generally conserve the scattering energy and the phase function asymmetry factor. Because of this, the normalization of the phase function has been proposed to guarantee that the scattering energy and the asymmetry factor are conserved. Various authors have used different normalization techniques. Three of these are compared in the present work, along with two other methods, one based on the finite volume method (FVM) and another one based on the spherical harmonics discrete ordinates method (SHDOM). In addition, the approximation of the Henyey–Greenstein phase function by a different one is investigated as an alternative to the phase function normalization. The approximate phase function is given by the sum of a Dirac delta function, which accounts for the forward scattering peak, and a smoother scaled phase function. In this study, these techniques are applied to three scalar radiative transfer test cases, namely a three-dimensional cubic domain with a purely scattering medium, an axisymmetric cylindrical enclosure containing an emitting–absorbing–scattering medium, and a three-dimensional transient problem with collimated irradiation. The present results show that accurate predictions are achieved for strongly forward scattering media when the phase function is normalized in such a way that both the scattered energy and the phase function asymmetry factor are conserved. The normalization of the phase function may be avoided using the FVM or the SHDOM to evaluate the in-scattering term of the radiative transfer equation. Both methods yield results whose accuracy is similar to that obtained using the DOM along with normalization of the phase function. Very satisfactory predictions were also achieved using the delta-M phase function, while the delta-Eddington phase function and the transport approximation may perform poorly.

1. Introduction

The discrete ordinates method (DOM) was initially developed by Chandrasekhar [1] in his work about interstellar radiation. A few years later, Carlson and Lathrop [2] applied the method to the neutron transport theory. There were some early attempts to use the method to solve thermal radiation problems, but it did not become popular until the last quarter of the 20th century, with the pioneering work of Fiveland [3]. Subsequent developments include improved spatial and angular discretization schemes, generalization to more complex grid structures and to complex enclosures, extensions to non-grey media, media with variable refractive index, and transient problems, as well as parallel implementation [4].

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Although the accuracy achieved using the DOM is often satisfactory and the computational requirements moderate, it is well known that errors due to ray effects and false scattering are generally present [5–7]. Another source of error arises in strongly anisotropic media, as a result of lack of conservation of scattered energy [8,9] and/or asymmetry factor of the phase function [10]. This is the subject of the present work, which is concentrated on scalar radiative transfer. Although the focus is on thermal radiation, the methods discussed here may be applied to other fields, for example atmospheric radiation, astrophysics, neutron transport, etc.

When the DOM is used, the in-scattering term of the radiative transfer equation (RTE) is approximated by a quadrature over all discrete directions, and there is no guarantee that the integral of the scattering phase function over a spherical surface of unity radius yields 4π, as it should if there were no approximation. A simple normalization of the scattering phase function to enforce conservation of the scattering energy has been proposed a long time ago by Kim and Lee [9] and implemented by Liu et al. [11]. However, it was found that this method only yields accurate results for small or moderate asymmetry factors of the phase function [10]. When the asymmetry factor increases, the normalization method changes the overall shape of the scattering phase function. As a consequence, even though the scattered energy is conserved, the asymmetry factor is not, and this may yield large errors in the case of highly anisotropic phase functions, which become larger in the case of optically thick media. Boulet et al. [12] found that this problem is largely overcome using the finite volume method (FVM) instead of the DOM. The effectiveness of the FVM to surmount this problem has also been found in problems with collimated irradiation [13].

A different normalization procedure, which conserves both the scattered energy and the asymmetry factor for the Henyey–Greenstein phase function, was proposed by Hunter and Guo [10]. This method produced results that closely agree with FVM predictions in an axisymmetric cylindrical enclosure [10], and Monte Carlo benchmark solutions in a cubic enclosure [14]. The application to a Legendre scattering phase function in ultrafast radiative transfer is reported in [15]. A drawback of this normalization procedure is the need to predetermine a normalization matrix. This may be avoided using a simpler normalization technique [16], which simultaneously conserves the scattered energy and the asymmetry factor of the phase function, and maintains most of the phase-function shape. The method was applied to the Henyey–Greenstein phase function, yielding results similar to those obtained using the previous normalization technique, but with a lower computational effort. This technique has recently been extended to collimated radiation for Henyey–Greenstein or Legendre scattering phase-functions [17].

It is also possible to approximate a strongly forward scattering phase function by another phase function where the forward scattering peak is dealt with a Dirac delta function, and the remaining scattering contribution is accounted for using an isotropic or a moderate anisotropic phase function. In such a case, the lack of conservation of scattered energy or asymmetry factor is largely overcome.

In the present work, the normalization methods of Liu et al. [11] and Hunter and Guo [10,16] are applied to three test problems along with two other methods that do not rely on the normalization of the phase function. One of these methods is the FVM, which is applied here only to the in-scattering term. This means that the DOM is still employed, except in the evaluation of that term, in contrast to previous works where the angular discretization of the RTE was fully carried out using the FVM. In this sense, the present approach is a DOM-FVM hybrid formulation. The other method is based on the spherical harmonics discrete ordinates method, which was proposed by Evans in the framework of atmospheric radiation [18]. We are not aware of previous application of this method to thermal radiation problems. The approximation of a strongly scattering forward phase function by a Dirac delta function plus a smooth scattering contribution is also investigated.

2. Mathematical formulation

The transient RTE for an emitting–absorbing–scattering grey medium may be written as follows [19]:

$$\frac{1}{c}\frac{\partial I(\mathbf{r},\mathbf{s},t)}{\partial t}+\mathbf{s}\cdot\nabla I(\mathbf{r},\mathbf{s},t)=-\beta\, I(\mathbf{r},\mathbf{s},t)+\kappa\, I_b(\mathbf{r})+\frac{\sigma_s}{4\,\pi}\int_{4\,\pi} I(\mathbf{r},\mathbf{s}',t)\;\Phi(\mathbf{s}',\mathbf{s})d\Omega' \tag{1}$$

where $I(\mathbf{r},\mathbf{s},t)$ is the radiation intensity in direction $\mathbf{s}$, $\mathbf{r}$ is the position vector, t is the time, c is the speed of light in vacuum, I_b is the blackbody radiation intensity, κ, β and σ_s are the absorption, extinction and scattering coefficients of the medium, respectively, and $\Phi(\mathbf{s}',\mathbf{s})$ is the scattering phase function. The boundary condition for a grey surface that emits and reflects diffusely is given by [19]:

$$I(\mathbf{r}_w,\mathbf{s},t)=\varepsilon\, I_b(\mathbf{r}_w)+\frac{\rho}{\pi}\int_{\mathbf{n}\cdot\mathbf{s}'<0} I(\mathbf{r}_w,\mathbf{s}',t)\left|\mathbf{n}\cdot\mathbf{s}'\right|d\Omega' \tag{2}$$

where $I(\mathbf{r}_w,\mathbf{s},t)$ and $I(\mathbf{r}_w,\mathbf{s}',t)$ are the radiation intensities at boundary point $\mathbf{r}_w$ that leave the boundary along $\mathbf{s}$ direction and arrive along $\mathbf{s}'$ direction, respectively, I_{bw} is the blackbody radiation intensity at the temperature of the boundary surface, ε is the surface emissivity, ρ is the surface reflectivity, and $\mathbf{n}$ is the unit vector normal to the surface and pointing into the medium. When collimated radiation is present, the RTE is decomposed into a diffuse and a collimated component, as described in Modest [19].

In the present work, the DOM was used to solve Eq. (1). The spatial discretization was performed using the CLAM scheme, and the angular discretization was carried out using the S_N quadrature, except in the cases identified below. In the case of transient problems, the time discretization was achieved using a second-order Runge–Kutta method. Details on the discretization procedure may be found elsewhere [20]. When the DOM is employed, the angular integrals are approximated by quadratures,