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The optical theorem for local source excitation of a particle near a plane interface

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ABSTRACT

Based on classic Maxwell's theory and the Gauss Theorem we extended the Optical Theorem to the case of a penetrable particle excited by a local source deposited near a plane interface. We demonstrate that the derived Extinction Cross-Section involves the total point source radiating cross-section and some definite integrals responsible for the scattering by the interface. The derived extinction cross-section can be employed to estimate the quantum yield and the optical antenna efficiency without computation of the absorption cross-section.

1. Introduction.

Many practical applications require analysis of electromagnetic scattering properties of local structures under point source excitation. Optical antennas have been introduced to enhance the energy transfer between a localized source and the detector or the free-radiation field. This enhancement can be achieved particularly by increasing the antenna efficiency. Surface plasmon resonances (PR) or localized PR make optical antennas particularly efficient at selected frequencies, which are important for biological sensing and detection [1,2]. Molecule fluorescence is important for the development of optical antennas employed in surface enhanced spectroscopy and microscopy [3]. Quantitative analysis of single-molecular fluorescence or photoluminescence of a quantum dot demands evaluation of the scattering efficiency of a local source in the presence of clusters of particles. Besides optimization of the antenna efficiency with respect to the emitter location, as well as averaging of enhancement factors under molecular fluorescent excitation requires multiple evaluation of the quantum yield for different polarizations and emitter locations [4,5]. Both the antenna efficiency and the quantum yield can be computed from the ratio of the scattered power P_{sc} to the total power $P_{sc}+P_{abs}$ including the absorbed power P_{abs} of the plasmonic structure $\eta = P_{sc}/(P_{sc}+P_{abs})$ [6]. Employing metal plasmonic nanostructures and PR excitation leads to the necessity to estimate the absorption cross-section of the structure at a PR regime when the relative field intensity near the plasmonic structure exceeds 10^8–10^{10} of magnitude [7,8]. All these circumstances require multiple computations of the energy flux over the elements of the plasmonic structure under investigation, which is needed to evaluate the absorption power P_{abs}. One a way to resolve this issue is to use the Optical Theorem (OT) to evaluate the antennas efficiency or the quantum yield η, which can be written in terms of scattering and extinction cross-sections only $\eta = C_{sc}/C_{ext}$.

The optical theorem has a long history; its occurrence in electromagnetic theory started more than one hundred years ago and similar theorems can be found in acoustical

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scattering and quantum mechanics [9]. The term OT has been well known ever since [10]. The OT is a powerful result in scattering theory, relating the extinction cross section of a structure to the scattering amplitude in the forward direction. Over the years, many derivations and implementations of the theorem have been provided [11–13]. In computational electromagnetics this theorem is particularly useful for checking or verification of the results of light scattering codes, since for a non absorbing particle, the integral of the scattered power over the total solid angle must be proportional to the imaginary part of the forward scattering amplitude [14,15]. The theorem has been reconsidered and generalized by a number of researchers to consider plane wave scattering by a particle near an interface between media with different refractive indices [16], and electromagnetic wave propagation in anisotropic media [17]. The OT was extended to the case of seismic wave propagation [18,19], as well as rough surfaces and beam excitation [20,21]. Excitation by a point sources on the other hand needs a new approach, and the OT has up to now been extended to point source excitation of a particle located in free space only [22,23].

In the present paper we extended the OT to the case of a penetrable particle excited by a point dipole deposited near a plane interface separating media with different refractive indices. We use classic Maxwell's theory and the Gauss Theorem as basic techniques.

2. Optical theorem for the local source on a plane interface.

Let us consider an excitation of a bounded penetrable particle D_i with a smooth surface ∂D_i by an electric dipole of momentum $\mathbf{p}$ deposited at a pointM_0, which is located outside ofD_i. For the considered scattering geometry see Fig. 1. Assume that the whole space $\mathbb{R}^3$ is constituted of two half-spaces $D_0(z>0)$ and $D_1(z<0)$ are separated by the plane interface$\Xi(z=0)$. Let the particle D_i be located inside the upper half-space $D_i \subset D_0$ and$M_0 \in D_0$. Then the mathematical statement of the scattering problem can be written in the form

$$\begin{gathered}\nabla\times\mathbf{H}_0 = jk\varepsilon_0\mathbf{E}_0 + \mathbf{p}\delta(M-M_0);\quad \nabla\times\mathbf{E}_0 = -jk\mu_0\mathbf{H}_0 \quad \text{in } D_0;\\ \nabla\times\mathbf{H}_l = jk\varepsilon_l\mathbf{E}_l;\quad \nabla\times\mathbf{E}_l = -jk\mu_l\mathbf{H}_l \quad \text{in } D_l,\quad l=01,i;\\ \begin{matrix}\mathbf{n}_q\times(\mathbf{E}_i(q)-\mathbf{E}_0(q))=0,\\ \mathbf{n}_q\times(\mathbf{H}_i(q)-\mathbf{H}_0(q))=0,\end{matrix}\ q\in\partial D_i,\quad \begin{matrix}\mathbf{e}_z\times(\mathbf{E}_1(Q)-\mathbf{E}_0(Q))=0,\\ \mathbf{e}_z\times(\mathbf{H}_1(Q)-\mathbf{H}_0(Q))=0,\end{matrix}\quad Q\in\Xi,\\ \lim_{r\to\infty}\int_{\Sigma_r}\left|\sqrt{\mu_l}\mathbf{H}_l\times\frac{\mathbf{r}}{r}-\sqrt{\varepsilon_l}\mathbf{E}_l\right|^2 d\sigma = 0,\quad r=|M|,\ l=0,1.\end{gathered}\tag{1}$$

here $\mathbf{n}_q$ - unit internal normal at ∂D_i, $\mathbf{e}_z$ - basic vector of the Cartesian coordinate system (x, y, z) orthogonal to Ξ and Σ_r - sphere of r - radius, centered at the plane Ξ. At infinity we use the Silver-Müller radiation conditions in week sense to avoid problem with surface waves [24]. Assume that$\partial D_i \subset C^{(2,\alpha)}$, ε_i, μ_i are constants inside D_i and$\text{Im}\varepsilon_i, \mu_i \le 0$, $\text{Im}\varepsilon_{0,1}, \mu_{0,1} = 0$. The time dependence was chosen as $\exp\{j\omega t\}$. Then the boundary value problem (1) has a unique solution [25].

Choose a sphere D_R of R - radius, centered at the plane Ξ and enclosing both D_i and point M_0 inside, its boundary will be referred to as Σ_R. The plane Ξ divides D_R by two half-spheres$D_R^{\pm}$, deposited in $D_{0,1}$, let $\Sigma_R^{\pm}$ be parts of Σ_R belonging to $D_R^{\pm}$ respectively. Applying the Gauss divergence theorem [26] to the solution of the problem (1) - fields$\mathbf{E}_0$ and $\mathbf{H}_0^*$ in the domain D_R^+/D_i we obtain

$$\begin{aligned}&\int_{D_R^+/D_i}\nabla\cdot\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]d\tau = \int_{D_R^+/D_i}\{\mathbf{H}_0^*\cdot\nabla\times\mathbf{E}_0 - \mathbf{E}_0\cdot\nabla\times\mathbf{H}_0^*\}d\tau\\ &= \int_{D_R^+/D_i}\left\{jk\left(-|\mathbf{H}_0^*|^2+|\mathbf{E}_0|^2\right)-(\mathbf{E}_0\cdot\mathbf{J}_0^*)\right\}d\tau\\ &= \int_{\Sigma_R^+\cup\Xi_R\cup\partial D_i}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot d\mathbf{s}\end{aligned}\tag{2}$$

where $\mathbf{J}_0 = \mathbf{p}\delta(M-M_0)$ and $\Xi_R :=\{M\in\Xi: |M|\le R\}$ is a part of the plane Ξ. Taking the real parts from both sides of (2) and rewriting the integrals in the right part we get

$$\begin{aligned}&\text{Re}\int_{\partial D_i}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot d\boldsymbol{\sigma} + \text{Re}\int_{\Sigma_R^+}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma\\ &-\text{Re}\int_{\Xi_R}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\mathbf{e}_z d\sigma = -\text{Re}\int_{D_R^+/D_i}(\mathbf{E}_e\cdot\mathbf{J}^*)d\tau\end{aligned}\tag{3}$$

Similar application of Gauss theorem inside D_i leads to

$$\text{Re}\int_{\partial D_i}\left[\mathbf{E}_i\times\mathbf{H}_i{}^*\right]\cdot d\boldsymbol{\sigma} = k\int_{D_i}\left\{|\text{Im}\mu_i||\mathbf{H}_i|^2+|\text{Im}\varepsilon_i||\mathbf{E}_i|^2\right\}d\tau\tag{4}$$

The right part of (4) we will refer as the absorption cross-section C_{abs}, then (3) accepts the form

$$\begin{aligned}&C_{abs}+\text{Re}\int_{\Sigma_R^+}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma - \text{Re}\int_{\Xi_R}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\mathbf{e}_z d\sigma\\ &= -\text{Re}\int_{D_R^+/D_i}(\mathbf{E}_0\cdot\mathbf{J}^*)d\tau\end{aligned}\tag{5}$$

Using Gauss theorem in the D_R^- domain and taking the real part we obtain

$$\text{Re}\int_{\Sigma_R^-}\left[\mathbf{E}_1\times\mathbf{H}_1^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma + \text{Re}\int_{\Xi_R}\left[\mathbf{E}_1\times\mathbf{H}_1^*\right]\cdot\mathbf{e}_z d\sigma = 0\tag{6}$$

Combining Eqs. (5) and (6) leads to

$$\begin{aligned}&C_{abs}+\text{Re}\int_{\Sigma_R^+}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma + \text{Re}\int_{\Sigma_R^-}\left[\mathbf{E}_1\times\mathbf{H}_1^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma\\ &= -\text{Re}\int_{D_R^+/D_i}(\mathbf{E}_0\cdot\mathbf{J}^*)d\tau\end{aligned}\tag{7}$$

We next consider the Far Field Patterns $\mathbf{F}_{0,1}(\theta,\varphi)$ of the fields [27] in the upper and lower half-spaces $D_R^{\pm}$

$$\mathbf{E}_{0,1}(M) = \frac{e^{-jkn_{0,1}r}}{r}\mathbf{F}_{0,1}(\theta,\varphi)+o\left(\frac{1}{r}\right),\quad r\to\infty,\quad z\ne 0.$$

The far field patterns are defined at unite hemi-spheres $\Omega^+ = \{0\le\varphi\le 360^\circ; 0\le\theta<90^\circ\}$, $\Omega^- = \{0\le\varphi\le 360^\circ;\ 90^\circ<\theta\le 180^\circ\}$ where $n_{0,1}=\sqrt{\varepsilon_{0,1}\mu_{0,1}}$. Because Eq. (7) is valid for any R we transform the integrals in the left side of Eq. (7) to integrals over the upper and lower hemi-spheres $\Sigma_R^{\pm}$ employing the radiation condition. It follows that

$$\begin{aligned}&\lim_{R\to\infty}\text{Re}\int_{\Sigma_R^+}\left[\mathbf{E}_0\times\mathbf{H}_0^*\right]\cdot\frac{\mathbf{r}}{r}d\sigma = \text{Re}\lim_{R\to\infty}\int_{\Sigma_R^+}\mathbf{E}_0\cdot\left[\mathbf{H}_0^*\times\frac{\mathbf{r}}{r}\right]d\sigma\\ &= \sqrt{\frac{\varepsilon_0}{\mu_0}}\lim_{R\to\infty}\int_{\Sigma_R^+}|\mathbf{E}_0|^2 d\sigma = \sqrt{\frac{\varepsilon_0}{\mu_0}}\int_{\Omega^+}|\mathbf{F}_0|^2 d\omega\end{aligned}$$