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Multiple scattering of a zero-order Bessel beam with arbitrary incidence by an aggregate of uniaxial anisotropic spheres

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ABSTRACT

Based on the generalized multiparticle Mie theory, multiple scattering of an aggregate of uniaxial anisotropic spheres illuminated by a zero-order Bessel beam (ZOBB) with arbitrary propagation direction is investigated. The particle size and configuration are arbitrary. The arbitrary incident Bessel beam is expanded in terms of spherical vector wave functions (SVWFs). Utilizing the vector addition theorem of SVWFs, interactive and total scattering coefficients are derived through the continuous boundary conditions on which the interaction of the particles is considered. The accuracy of the theory and codes are verified by comparing results with those obtained for arbitrary plane wave incidence by CST simulation, and for ZOBB incidence by a numerical method. The effects of angle of incidence, pseudo-polarization angle, half-conical angle, beam center position, and permittivity tensor elements on the radar cross sections (RCSs) of several types of collective uniaxial anisotropic spheres, such as a linear chain, a $4 \times 4 \times 4$ cube-shaped array, and other periodical structures consisting of massive spheres, are numerically analyzed. Selected results on the properties of typical particles such as TiO_2, SiO_2, or other particle lattices are calculated. This investigation could provide an effective test for further research on the scattering characteristics of an aggregate of anisotropic spheres by a high-order Bessel vortex beam. The results have important application in optical tweezers and particle manipulation.

1. Introduction

Electromagnetic interactions with anisotropic materials have recently become a subject of great interest because of the increasing number of technological and biological applications of these materials. Given that the permittivity and permeability are characterized by tensors with 2 nonzero tensor elements for uniaxial anisotropic medium, 4 different nonzero tensor elements for plasma anisotropic medium, and 9 nonzero tensor elements for arbitrary anisotropic media, the vector wave equations of EM fields are coupled. Obtaining the solution of the wave equations, which is distinct from that in isotropic media is difficult. Two primary analytical methods to represent EM fields in the anisotropic medium in spherical coordinate system over the last several decades. The first of these methods uses two scalar wave functions consisting of fractional-order Riccati-Bessel functions to describe vector fields. Using this method, Wong [1], Liu [2] and Qiu [3–5] expanded the EM fields in a rotationally uniaxial anisotropic sphere in terms of the modified SVWFs **M** and **N**, and analyzed the scattering characteristics illuminated by a plane wave. Wang et al. [6] investigated the scattering of a rotationally uniaxial anisotropic sphere illuminated by a Gaussian beam. The second method is called the Fourier transformation method. Using this method, Geng [7,8]

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expanded the EM fields in a uniaxial and plasma anisotropic sphere in terms of the whole SVWFs **M**, **N** and **L**, and studied the scattering of a uniaxial and plasma anisotropic sphere from a plane wave. Following the work, Wu et al. [9,10] extended the interaction between a uniaxial anisotropic sphere with a plane wave to with a Gaussian beam. Using differential theory, Stout et al. [11] presented the solution of scattering of an arbitrary-shaped body made of arbitrary anisotropic medium, but did not provide numerical results. Some numerical methods such as the finite-difference time domain method [12], the moment method [13], and the Null-field method (otherwise known as T-Matrix method et al.) [14], have been used to study the scattering problem. However, the scatterer in all these studies is limited to a single anisotropic particle. Some numerical method such as the Null-field method proposed by Adrian et al. [15] and the Multiple multipoles (MMP) proposed by Christian [16] may be applicable to the multiple scattering of anisotropic particles. However, investigations on multiple scattering of a cluster of anisotropic particles using an analytical approach are limited.

The problem of multiple scattering by an aggregate of particles is important because of the wide range of potential applications. After the addition theorem of SVWFs was put forward [17,18] and developed by Mackowski [19] and Xu [20], the rigorous analytical solution of multiple scattering by an ensemble of isotropic spherical particles was obtained and extensively developed. Bruning and Lo [21] first presented a comprehensive solution of scattering by a two-sphere chain. Fuller and Kattawar [22] obtained the consummate solution of EM scattering by a cluster of spheres. Xu introduced the GMM to explore EM scattering by an aggregate of isotropic spheres [23,24] and investigated multiple scattering by multiple isotropic spheres. Based on GMM and generalized Lorenz–Mie theory (GLMT), Gouesbet et al. derived the analytical solution of the scattering of assemblies of spheres and aggregates illuminated by an arbitrary electromagnetic shaped beam [25]. The scattering characteristics of soot aggregate particles [26,27] and periodic particle arrays [28,29] have recently been investigated. Numerical methods such as the null-field method [30–32] and discrete-dipole approximation (DDA) [33] are also effective solutions to this problem. However, in most of related literatues, the investigations focus on plane wave scattering by multiple isotropic spheres. Recently, we have investigated the EM scattering of multiple uniaxial anisotropic spheres by a plane wave [34,35] and Gaussian beam [36].

Nevertheless, the multiple scattering of a Bessel beam by an aggregate of anisotropic particles has not yet been reported. In fact, Bessel beams have attracted widespread attention in various fields, such as optical trapping and manipulation, particle sizing, optical guiding and alignment [37–42] since its naissance by Durnin [43], because of its non-diffraction and self-reconstruction properties. Using EM field representation, the scattering problem of a Bessel beam by spherical particles has been investigated by several scholars. Ma and Li [44] investigated unpolarized Bessel beam scattering by a dielectric sphere, and derived the corresponding dimensionless scattering functions. Using GLMT [45], Li et al. [46] investigated the scattering of a spherical particle illuminated by an axicon-generated Bessel beam. Ambrosio [39] also provided an expansion coefficient in terms of SVWFs by applying an integral localized approximation. Mishra [47] derived a vector wave theory to describe the Bessel beam for an arbitrary half-conical angle. Using this vector description, Mitri investigated the scattering of a ZOBB [48] and a high-order Bessel vortex beam [49] by a dielectric sphere and evaluated the radial electric fields decaying in proportion to $(1/r)^2$ by plotting 3D-directivity patterns. Mitri et al. [50] recently presented a complete description of vortex vector Bessel beams in the context of GLMT for EM resonance scattering by a dielectric sphere. Using an analytical method, the scattering of a ZOBB by a concentric sphere was investigated by Chen et al. [51], but this ZOBB was assumed to present on-axis incidence only. Utilizing a numerical method, Cui et al. studied the scattering of a ZOBB by a random discrete [52] homogeneous dielectric particle. Despite the wealth of knowledge obtained from these works, however, however, the scatterer in all of these previous studies is limited to isotropic particles.

We recently investigated the scattering of an off-axis ZOBB by a uniaxial anisotropic sphere [53] and uniaxial anisotropic bispheres [54]. In our work, the incident direction of the Bessel beam is parallel to the primary optical axis of the uniaxial anisotropic sphere, and the expansion coefficients of the off-axis Bessel beam are given. When the incident beam is not parallel to the primary optical axis, the scattering problem becomes complicated, and several unique scattering characteristics are observed. Given that the Bessel beam exhibits a special trait in comparison with the plane wave and Gaussian beam, describing the interaction of an aggregate of uniaxial anisotropic spheres with an arbitrary configuration and a Bessel beam with an arbitrary propagation direction is an important endeavor. As the ZOBB can easily be realized in the laboratory, we consider the scattering problem of a ZOBB.

In this paper, we focus on the interaction of a ZOBB and an aggregate of uniaxial anisotropic spherical particles. Specifically, the incident Bessel beam is described by a vector expression with arbitrarily propagation direction. In Section 2, the arbitrary incident ZOBB is presented with Euler rotation angles. Moreover, the expansion expression and coefficients of the incident field in terms of the SVWFs are given, and multiple scattering theory of anisotropic spheres is shown, and the analytical solution is derived. Finally, far-field scattering is described. Section 3 presents the numerical results and discussions and Section 4 presents our conclusion. In addition, a time dependence of the form $\exp(-i\omega t)$ is assumed and suppressed in the subsequent depiction, where ω is the circular frequency.

2. Theoretical analysis

A Cartesian coordinate system *Oxyz* is built with a fixed global coordinate system to indicate the randomness of the incident direction of Bessel beam and the configuration of the aggregation of particles [Fig. 1(a)]. Considering L uniaxial anisotropic spheres with radius a_j ($j=1,2,\ldots,L$) and