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Analysis of the far-field characteristics of hybridly polarized vector beams from the vectorial structure

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ABSTRACT

Based on the angular spectrum representation of electromagnetic beams, analytical expressions are derived for the TE term, TM term and the whole energy fluxes of a hybridly polarized vector (HPV) beam propagating in the far field. It is shown that both the TE and TM terms of the energy fluxes are strongly dependent of the truncation radius of the circular aperture. By choosing the truncation radius as a certain value, it is found that the far-zone distributions of TE and TM terms exhibit four-petal patterns with surrounding side-lobes displaying oscillating intensities. Interestingly, such phenomenon becomes extremely obvious particularly when the truncation radius is comparable with the wavelength of the propagating beam.

1. Introduction

Non-uniformly polarized (NP) laser beams, which can be generated from either radially or azimuthally polarized (AP) beams by utilizing a liquid crystal variable retarder (LCVR) [1], attracted substantial interests in the past two decades due to their uniquely elliptical polarization. By way of illustration, the incoherent NP beam array has the capacity to reduce the turbulence-induced scintillations in free space optical communication (FSOC) channels [2]. Cylindrically symmetry polarization of laser beams (also referred as the cylindrically polarized vector (CV) beams), which can be generated by a NP beam array, were extensively studied because of their potential applications in the polarization encoding, polarimetry and high-resolution imaging [3]. Propagation properties of cylindrically polarized, partially coherent beams in free space were investigated [4,5]. Analytical expressions for azimuthally and radially polarized fields were derived based on the angular spectrum representation [6]. An experimental method was introduced to generate AP beams with arbitrary spatial coherence [7,8]. Global parameters were introduced to describe radial and azimuthal polarization states of a NP beam [9,10], respectively. In addition, propagation properties of radially and AP beams in free space [11–13] and isotropic turbulence [14] were intensively studied over the past few years, respectively. It was found that both radially and AP beams can be generated by using various approaches, which include the optical fibers with fabricated sub-wavelength concentric metallic gratings [15], the coherent polarization beam combination of LP11 fiber modes [16] and a two-mode etched fiber [17].

Apart from above researches, radial-variant vector fields with hybrid states of polarization (SOPs) were also paid close attentions as they have great potentials in optical trapping experiments [18]. It was shown that either a super-length optical needle along the optical axis, or a sub-diffraction

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beam pattern can be generated by the tight focusing of a hybridly polarized vector (HPV) beam through a dielectric interface of annular NA-lens [19]. Furthermore, the increment of the radial index of NA-lens is capable to induce changes of focal patterns of a HPV beam from an elliptical ring to a circular spot [20,21]. Experimental setup was devised to generate arbitrary polarized HPV beams by using a radially polarized light transmits through a sequence of wave plates and SLM [22,23].

To date, however, internal structural properties of a HPV laser beam in the far field, to the best of our knowledge, have not been addressed in any literature. Besides, it is important to know the TE and TM distributions of energy fluxes of a HPV beam in a quantitative point of view. To this end, we particularly design the following procedures: 1) Based on the angular spectrum representation, analytical expressions are derived for the TE term, TM term and the whole energy flux distributions of the HPV beam; 2) Numerical simulations are performed to study vectorial structural properties of the HPV beam in the far-field; 3) Some interesting phenomena of the propagating beam are discussed and the primary results are summarized.

2. Internal vectorial structure of a HPV beam in the far field

Suppose that a HPV beam propagates toward the half-space $z \ge 0$ along the z-axis. The beam has a hybrid state of polarization (SOP), which can exhibit linear, elliptical, and circular polarization states. Typically, the electric field of the HPV beam can be alternatively expressed as [19,20]:

$$\vec{\mathbf{E}}(r') = A_0 circ(r'/R_0)[\cos(2n_0\pi r'/R_0+\alpha)\vec{\mathbf{e}}_\mathbf{x} + \sin(2n_0\pi r'/R_0+\alpha)\vec{\mathbf{e}}_\mathbf{y}], \tag{1}$$

or

$$\vec{\mathbf{E}}(r') = A_0 circ(r'/R_0)\{\exp[i(2n_0\pi r'/R_0+\alpha)]\vec{\mathbf{e}}_\mathbf{x} + \exp[-i(2n_0\pi r'/R_0+\alpha)]\vec{\mathbf{e}}_\mathbf{y}\}, \tag{2}$$

where A_0 is a constant, $r'=(x',y')$ represents a spatial variable within the transversal plane $z=0$. $\vec{\mathbf{e}}_\mathbf{x}$ and $\vec{\mathbf{e}}_\mathbf{y}$ are the unit vectors in the Cartesian coordinate, R_0 denotes the truncation radius of a circular aperture, n_0 represents the radial index which determines the SOPs of the beam, $circ(r/R_0)$ is the circular function [24,25]. $\alpha = \arctan(y'/x')$ is the phase of the beam at the initial plane $z=0$. According to the vectorial angular spectrum representation of electromagnetic beams [26,27], the electric field of the propagating HPV beam can be expressed as:

$$\vec{\mathbf{E}}(r) = \int\int_{-\infty}^{+\infty} \{A_x(p,q)\vec{\mathbf{e}}_\mathbf{x} + A_y(p,q)\vec{\mathbf{e}}_\mathbf{y} - \frac{\vec{\mathbf{e}}_\mathbf{z}}{m}[pA_x(p,q)+qA_y(p,q)]\} \times \exp[ik(px+qy+mz)]dpdq, \tag{3}$$

where $r = x\vec{\mathbf{e}}_\mathbf{x} + y\vec{\mathbf{e}}_\mathbf{y} + z\vec{\mathbf{e}}_\mathbf{z}$ represents a spatial variable within the output plane z, $\vec{\mathbf{e}}_\mathbf{z}$ is the unit vector which denotes the propagation direction of the beam. In the far field, evanescent waves hardly contribute to the propagating field, therefore, $m = (1-p^2-q^2)^{1/2}$ is fulfilled in Eq. (3). $A_x(p,q)$ and $A_y(p,q)$ are the Fourier transforms of the electric field components $\vec{\mathbf{E}}_\mathbf{x}(x',y')$ and $\vec{\mathbf{E}}_\mathbf{y}(x',y')$, respectively:

$$A_x(p,q) = \frac{1}{\lambda^2}\int\int_{-\infty}^{+\infty} \vec{\mathbf{E}}_\mathbf{x}(x',y',0)\exp[-ik(px'+qy')]dx'dy', \tag{4}$$

$$A_y(p,q) = \frac{1}{\lambda^2}\int\int_{-\infty}^{+\infty} \vec{\mathbf{E}}_\mathbf{y}(x',y',0)\exp[-ik(px'+qy')]dx'dy', \tag{5}$$

where λ is the beam wavelength. In Eqs. (1) and (2), the circular function can be given as an expansion of a linear summation of Gaussian functions [24]:

$$circ(r'/R_0) = \sum_{n=1}^{N} A_n \exp(-B_n r'^2/R_0^2), \tag{6}$$

where the coefficients A_n and B_n can be indexed in [25]. Then the Fourier transform of the field component along the x-axis can be obtained by substituting Eq. (2) into (4):

$$\begin{aligned} A_x(p,q) &= \frac{A_0}{\lambda^2}\exp[i\arctan(q/p)] \times \int_0^\infty \sum_{n=1}^{N} A_n \exp(-B_n r'^2/R_0^2) E_x(r') J_1(kbr) r dr \\ &= \frac{A_0}{\lambda^2}\exp[\arctan(q/p)] \sum_{n=1}^{N} A_n \sum_{m=0}^{\infty}\left(i\frac{2n_0\pi}{R_0}\right)^m \times \int_0^\infty r'^{m+1}\exp(-B_n r'^2/R_0^2) J_1(kbr')dr', \end{aligned} \tag{7}$$

where $b=(p^2+q^2)^{1/2}$. It is worthwhile to note that the integration in Eq. (7) can be performed by using the following formula [28]:

$$\int_0^\infty \exp(-\alpha\rho^2) J_n(\beta\rho)\rho^{m-1}d\rho = \frac{1}{2}\alpha^{m/2}\left(\frac{\beta^2}{4\alpha}\right)^{n/2}\frac{\Gamma\left(\frac{m+n}{2}\right)}{n!}\exp\left(-\frac{\beta^2}{4\alpha}\right) {}_1F_1\left(\frac{n-m}{2}+1; n+1; \frac{\beta^2}{4\alpha}\right), \tag{8}$$

where $\Gamma(.)$ is the Gamma function, ${}_1F_1(a;b+1;z)$ denotes the Kummer function which can be further expanded into power series:

$${}_1F_1(a,b+1;z) = \frac{b!}{\Gamma(a)}\sum_{s=0}^{\infty}\frac{\Gamma(a+s)z^s}{(b+s)!s!}. \tag{9}$$

Based on Eq. (9), the Fourier transform $A_x(p,q)$ can be derived:

$$A_x(p,q) = \frac{\pi A_0 R_0^3}{2\lambda^3} b\exp[i\arctan(q/p)] \sum_{n=1}^{N} A_n \sum_{m=0}^{\infty}(i2n_0\pi)^m B_n^{-\frac{m+3}{2}}\Gamma\left(\frac{m+3}{2}\right) \times {}_1F_1\left(\frac{m+3}{2}; 2; -\frac{k^2R_0^2}{4B_n}b^2\right). \tag{10}$$