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# Surprises and anomalies in acoustical and optical scattering and radiation forces

Philip L. Marston

*Physics & Astronomy Department, Washington State University, Pullman, WA 99164-2814, USA*

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ABSTRACT

Experiments on radiation torques and negative radiation forces by various researchers display how the underlying wave-field geometry influences radiation forces. Other situations strongly influenced by wave-field geometry include high-order caustics present in light-scattering patterns of objects as simple as oblate drops of water or oblate bubbles of air in water. Related theoretical and experimental investigations are considered. Acoustic scattering enhancements associated with various guided waves are also examined. These include guided waves having negative group velocities and guided wave radiating wavefronts having a vanishing Gaussian curvature.



## 1. Introduction

Summarized here are examples of scattering research from the past 35 years. Some of the research became perhaps of greater interest during the past decade than when the original investigations were carried out. In the period leading up to these investigations, the author was significantly influenced by van de Hultst's physically oriented monograph *Light Scattering by Small Particles*. It is appropriate to jointly consider acoustic and optical wave processes since both types of time-harmonic wave fields are usually described using the Helmholtz equation. Some amount of similarity between sound and light is suggested historically by classical Greek words related to light and sound: φωτος and φωνη (related to the modern terms *photo* and *phono*). Noted in Appendix A is supplemental information pertaining to historical background and selected recent developments. Readers are encouraged to consult the publications referenced there and in the body of the paper for the broader context.

## 2. Optical angular momentum and torque

In about 1900 Sadowsky (and later Poynting) realized that circularly polarized plane waves should exert a torque in certain situations. Beth confirmed this in the 1930s. In 1983, the present author noticed that an analysis of the torque on an isotropic sphere predicted by the Mie theory was unavailable. If the incident circularly polarized light has positive helicity, the torque (using the time-averaged Maxwell stress evaluated in the far field) was found to be [1]: $\Gamma = P_{abs}/\omega$, where $P_{abs}$ is the absorbed power. The various special cases in [1] considered including enhancing $\Gamma$ and $P_{abs}$ by selecting $\omega$ to excite a surface plasmon mode of a small metallic sphere. This enhancement was recently used to spin gold nano-spheres in water at an angular frequency of a few kHz [2]. It may appear surprising that there is no torque on an isotropic non-absorbing sphere. To understand this, Marston and Crichton [3] used

*E-mail address:* marston@wsu.edu

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Humblet's decomposition of field angular momentum (from the 1940s) to analyze the angular momentum carried by the scattered radiation. Part of the axial component of angular momentum of the scattered field is attributed to "spin" and is associated with part of the scattered power:

$$P_{spin} = \int (I_+ - I_-) \cos\theta\, r^2\, d\Omega \quad (1)$$

where $I_+$ and $I_-$ denote positive and negative helicity components, respectively, of the far-field scattered irradiance evaluated at a scattering angle $\theta$, $r$ is the distance to the observer, and the integration is over a solid angle of $4\pi sr$. The difference $(I_+ - I_-)$ corresponds to a Stokes parameter [4] and the factor $\cos\theta$ is needed to obtain the local axial projection. Evaluating $I_+$ and $I_-$ using the Mie theory and introducing the spin efficiency factor $Q_{spin}$ where $P_{spin} = I_0 \pi a^2 Q_{spin}$ gives [3,4]

$$Q_{spin} = \left[2/(ka)^2\right] \sum_{n=1}^{\infty} \left( \left\{ \left( |a_n|^2 + |b_n|^2 \right) (2n+1)/[n(n+1)] \right\}. + \left\{ [2n(n+2)/(n+1)] \mathrm{Re}[a_{n+1} b_n^* + a_n b_{n+1}^*] \right\} \right), \quad (2)$$

where $I_0$ is the illumination irradiance, $a$ is the sphere radius, $k = \omega/c$, and $a_n$ and $b_n$ are Mie coefficients in the usual notation [1]. The Humblet decomposition shows that there is an axial projection of the flux of *orbital* angular momentum having an efficiency factor $(Q_{sca} - Q_{spin})$ where $Q_{sca}$ is the scattering efficiency [3,4]. Omitting the orbital angular momentum gives the incorrect impression of a torque on lossless isotropic spheres illuminated by circularly polarized light.

While the aforementioned result $\Gamma = P_{abs}/\omega$ pertains to isotropic spheres, even prior to the 1980s there was significant interest pertaining to optical torques associated with other scattering situations. References concerned with the optical windmill torque on non-spherical particles are noted in [1].

## 3. Negative radiation forces and scattering by Bessel beams

Much 20th century research on scattering by *acoustic* beams concerned situations involving low frequencies (associated with monopole and dipole scattering terms) or situations in which the description of the beam was valid only at high frequencies. Subsequently Durnin's angular spectrum representation was used to express the scattering by an arbitrary isotropic sphere centered on the axis of a Bessel beam [5,6]. The scattering is expressed in terms of partial wave coefficients $(s_n - 1)$ for the corresponding *plane wave* scattering problem with a modified weighting factor that depends on the conic angle $\beta$ of the Bessel beam. The $(s_n - 1)$ notation for partial wave coefficients in acoustics was widely used in the 1970s and 1980s for the same reasons analogous notation was earlier found to be advantageous in the quantum theory of scattering. For Bessel beam illumination it was then possible to compute the scattering properties and the associated acoustic radiation force for a variety of spheres by appropriate selection of the $s_n$ from prior results for plane waves [5–8]. In the investigation of scattering of Bessel beams by spheres, a background subtraction method was used to formally extract elastic scattering contributions, a procedure often identified with the "theory of resonance scattering" [7]. Conditions were introduced for suppressing the excitation of specific partial-waves through the appropriate selection of $\beta$ for spheres centered on ordinary and *vortex* Bessel beams [7,9].

Situations were discovered [5,8] for appropriate values of $\beta$ and material properties such that the direction of radiation force was predicted to be opposite the propagation direction of the beam. These results were extended to the case of spheres centered on the axis of a first-order Bessel *vortex* beam [9,10]. For both kinds of beams, negative radiation forces were associated with situations in which scattering into the backward hemisphere was greatly suppressed relative to the scattering into the forward hemisphere. (Mitri [11] and others provided other related derivations. Negative force situations are sometimes referred to as "tractor beams".) Consideration of a partially analogous optical situation [12] provided important insight into the momentum projected by a Bessel beam. Subsequently Zhang and Marston carried out a corresponding geometric analysis of the radiation force on spheres centered on the axis of an $m$th order acoustic Bessel beam giving a radiation force proportional to the following dimensionless function [13]:

$$Y_p = (b - \langle w \rangle) Q_{sca} + b Q_{abs}, \quad (3)$$

where $Q_{sca}$ and $Q_{abs}$ are dimensionless scattering and absorption efficiencies and $\langle w \rangle$ is the scattering asymmetry (defined by extending Van de Hulst's terminology to the present situation) and $b = \cos\beta$ where $\beta$ is the conic angle of the Bessel beam. The asymmetry $\langle w \rangle$ satisfies the condition $-1 \le \langle w \rangle \le 1$. Inspection of (3) shows that $Y_p$ and the corresponding force is negative only when $\langle w \rangle$ is sufficiently positive and $\beta$ is sufficiently large. In (3), all of the factors depend implicitly on $\beta$. Though (3) was analytically derived in [13], it is consistent with the geometrical interpretation in [12]. The geometrical interpretation is shown in Fig. 1 and is based on momentum projection. (Similar projections have been used to demonstrate negative radiation forces for other types of optical [14] and acoustic [15,16] beams.) Consider first the situation where a sphere centered at O on the beam's axis has no energy absorption. When the scattering angle $\theta$ is less

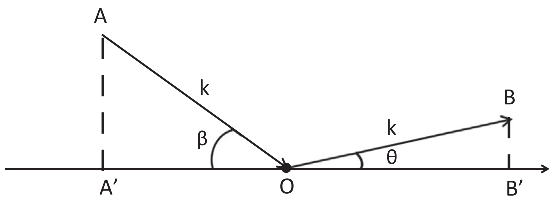


**Fig. 1.** The diagram shows wave vector projections for a sphere centered on a Bessel beam having a conic angle $\beta$. The diagram is relevant to situations producing negative optical or acoustical radiation forces on the sphere. When the scattering angle $\theta$ is less than $\beta$, the axial projection of the scattered wave vector exceeds the axial projection of the incident wave vector. From momentum conservation such processes can produce negative axial radiation forces if the positive force processes are sufficiently small [12,13].