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A persistent feature of multiple scattering of waves in the time-domain: A tutorial

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ABSTRACT

The equations for frequency-domain multiple scattering are derived for a scalar or electromagnetic plane wave incident on a collection of particles at known positions, and in the time-domain for a plane wave pulse incident on the same collection of particles. The calculation is carried out for five different combinations of wave types and particle types of increasing geometrical complexity. The results are used to illustrate and discuss a number of physical and mathematical characteristics of multiple scattering in the frequency- and time-domains. We argue that frequency-domain multiple scattering is a purely mathematical construct since there is no temporal sequencing information in the frequency-domain equations and since the multi-particle path information can be dispelled by writing the equations in another mathematical form. However, multiple scattering becomes a definite physical phenomenon in the time-domain when the collection of particles is illuminated by an appropriately short localized pulse.

1. Introduction

The subject of this tutorial is the physical and mathematical interpretation of the equations of multiple scattering of waves in the frequency- and time-domains. Martin (see Sec. 1.1 of [1]) has correctly remarked that "multiple scattering" may mean different things to different scientists. We should add that the meaning of the term "multiple scattering" can also vary depending on the particular context in which it appears. More often than not, this has to do with the fact that although physics describes actual natural phenomena, it does so using the abstract language of mathematics. It is therefore important to delineate the situations when multiple scattering represents a real physical process and those when it emerges as a purely mathematical construct.

In the frequency-domain, a monochromatic plane wave or shaped beam of infinite temporal duration is incident on a collection of scattering particles at known fixed positions. Five attributes of multiple scattering of waves in the *frequency-domain* have recently been described in [2–5] and pp. 6–9, 47, 65–66 of [6] in order to clarify a number of common misconceptions. They are as follows.

(i) If one wishes to discuss wave scattering by a collection of many particles in terms of multiple scattering, the so-called compact form of the pertinent equations can be expanded, purely mathematically, as a particular infinite series of terms known as the expanded form of the equations [7]. This series has been called the multiple-scattering point of view [8], an order-of-scattering expansion [9–11], or a multi-path expansion of the total wave (see [7] and pp. 765–766 of [12]).
(ii) The multiple-scattering point of view in the frequency-domain with the incident wave being scattered sequentially by one, two, three, or more particles before reaching the observation point does not refer to an actual physical phenomenon. It is only a purely mathematical expansion of the total wave. This is because in the frequency-domain

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all the mutual excitations occur simultaneously and are not temporally discrete and ordered events.

(iii) The scattered wave leaving particle *i* to be rescattered by particle *j* does not propagate only in the specific direction from *i* to *j* as is the case for a multiply-scattered projectile particle.

(iv) For frequency-domain scattering in general, the scattering equations are consistent with the point of view that the wave incident on particle *j* is not transformed into or replaced by the scattered wave. Thus the cause of scattering at particle *j* is not the incident field, but rather the presence of an object with physical properties different from that of the external medium.

(v) The form of the total field in the exterior medium makes it clear that both the incident and scattered fields, considered individually, are purely mathematical entities rather than actual physical objects. The only actual physical object is the total field, either in the presence of or in the absence of the scattering particle. The scattered field is defined to be the difference between the total field in the exterior region with and without the scatterer present.

To demonstrate that the situation can be profoundly different in the time-domain, in the body of this tutorial we describe exactly soluble time-domain multiple scattering scenarios involving an incident narrow Gaussian plane wave pulse. We obtain the solutions for five fixed (deterministic) scattering objects of increasing geometric complexity and analyze them using four different mathematical approaches.[1] The examples considered here are used to revisit, and refine where necessary, the above *frequency-domain* statements (i)–(v) formulated for an incident beam of infinite temporal duration. They are also used to extend some of the statements when one considers *time-domain* scattering of a temporally-short incident pulse. The specific selection of the five scattering scenarios of increasing complexity is intended to maximize the pedagogical value of our discussion for the reader.

We start with the simple examples of one-dimensional scattering, and then isotropic scattering of scalar waves in three dimensions. Then we progress to the more complicated and realistic situations of particles of finite size, arbitrary shape and internal composition, and incident electromagnetic waves rather than scalar waves. As the examples become more elaborate and complicated, the mathematics and notation needed to fully and accurately describe them become more elaborate and complicated as well. This is why we consider the simplest geometries first, in order to avoid the complexity of the notation from obscuring the patterns that occur in the equations and the points we are trying to illustrate.

In spite of the increased complexity as we progress from example to example, certain time-domain multiple-scattering phenomena occur in exactly the same way time after time, i.e. they persist. Specifically, scattering of a temporally-short pulse results in a temporal succession of distinct individual scattered pulses in a one-to-one correspondence with the different multi-particle paths encountered in the frequency-domain. This result is obtained by first separating out all the rapidly-varying wave number-dependent phases in the terms of the expanded form of the frequency-domain multiple scattering equations. The mathematical details of this separation turn out to be different for the different examples considered. The resulting temporal succession of pulses obtained in the time-domain is not valid for only the simplest of examples, but persists as the geometrical complexity of the system increases. As a result, the pulse sequence has the effect of elevating multiple scattering from being an abstract mathematical entity in the *frequency-domain* to an actual physical phenomenon in the *time-domain*.

2. General considerations

Before describing the five multiple scattering scenarios in detail, it is instructive to discuss the definition of a number of terms used throughout this tutorial. The first of these, as in statement (v) of Section 1, is the difference between purely mathematical entities and actual physical objects. In Lorenz–Mie scattering a linearly polarized monochromatic electromagnetic plane wave having infinite transverse extent and infinite temporal duration, angular frequency ω, wavelength λ, and wave-number $k=2\pi/\lambda$ is incident on a single spherical particle of radius a and refractive index N whose center is at the origin of coordinates. The scalar radiation potential (see Sec. 9.21 of [13]) of the total wave exterior to the particle satisfies the scalar wave equation subject to the appropriate boundary conditions, and is standardly chosen to have the form

$$\psi_{total}(k\mathbf{r})\exp(-i\omega t)=\psi_{beam}(k\mathbf{r})\exp(-i\omega t)+\psi_{scatt}(k\mathbf{r})\exp(-i\omega t), \quad (1)$$

where the electromagnetic fields of the incident and scattered waves are obtained by vector differentiation of ψ_{beam} and ψ_{scatt}, respectively. In Eq. (1), $\mathbf{r}$ is the position vector, t is time, and $i=(-1)^{1/2}$. The motivation of the assumption that the incident wave is present in the entirety of the exterior region is due to the fact that the portion of it that is transversely distant from the scattering particle passes from upstream locations to downstream locations without ever directly encountering the particle or being diffracted by it. It was stated in [2–6] that the only wave with a definite physical existence in this situation is ψ_{total}, and the decomposition of ψ_{total} into the sum of ψ_{beam} and ψ_{scatt} is a purely mathematical construction.

This is an instance of a more fundamental problem: if a wave ψ_{total} is decomposed into the sum of two hypothetical parts ψ_1 and ψ_2,

$$\psi_{total}=\psi_1+\psi_2, \quad (2)$$

when is such a decomposition purely an abstract mathematical construction and when do the two parts have a definite physical existence? The answer depends crucially on the details of the experimental configuration and the attributes of the two hypothetical parts. If an experiment

[1] We note that stochastic scattering objects in the form of a group of particles located at statistically random positions involve a number of issues that will not be discussed here. Moving scatterers will also not be considered, since their analysis lies in the realm of dynamic light scattering.