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Direct collocation meshless method for vector radiative transfer in scattering media

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A B S T R A C T

A direct collocation meshless method based on a moving least-squares approximation is presented to solve polarized radiative transfer in scattering media. Contrasted with methods such as the finite volume and finite element methods that rely on mesh structures (e.g. elements, faces and sides), meshless methods utilize an approximation space based only on the scattered nodes, and no predefined nodal connectivity is required. Several classical cases are examined to verify the numerical performance of the method, including polarized radiative transfer in atmospheric aerosols and clouds with phase functions that are highly elongated in the forward direction. Numerical results show that the collocation meshless method is accurate, flexible and effective in solving one-dimensional polarized radiative transfer in scattering media. Finally, a two-dimensional case of polarized radiative transfer is investigated and analyzed.

1. Introduction

Solar light propagating through the atmosphere, the ocean, and vegetation is often analyzed in the framework of scalar radiative transfer theory. However, due to the electromagnetic nature of light, the effects of polarization should be considered in the treatment of scattering, reflection, and transmission [1,2]. The interaction of light with media, such as scattering by particles and reflection at phase boundaries, will generally alter the polarization of the incident beam [3,4]. Polarization is also decoupled from the frequency of the light, giving extra degrees of freedom. Accurate and efficient polarized radiative transfer calculations are essential in many applications, such as the retrieval of atmospheric and oceanic constituents from remotely sensed satellite observations [5]. For skylight detection, the additional polarimetric measurements can significantly improve the retrieval of some aerosol parameters, and several space-borne and airborne instruments have been designed to measure the polarization of skylight. This also has potential applications in other disciplines, such as biomedical optical tomography [6]. Many applications in medical diagnostics profit from polarization properties, e.g. noninvasive glucose sensing, tissue anisotropy and concentration measurements.

Considering the importance of polarization information, polarized radiative transfer is an active area of research. The standard vector radiative transfer equation (VRTE), which is applied to the propagation of light in a plane-parallel layer, has been utilized for more than half a century [7]. To investigate the polarization characteristics in graded index media, a corresponding VRTE was derived [8]. Recently, for astrophysical applications, a VRTE in an orthogonal curvilinear coordinate system was derived [9]. Many numerical methods have been proposed and developed to investigate polarized radiative transfer, including Monte Carlo (MC) methods [10–14], F_N methods [15], discrete ordinate (DO)

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Nomenclature

A	stiffness matrix
a	coefficient matrix
a	expansion coefficient of Legendre polynomial
B	source matrix
d	diameter of the support domain, m
F	incident light irradiance, W/m^2 sr
H	height, m
Δh	local discrete length, m
I	radiation intensity, W/m^2 sr
i, **j**, **k**	unit vectors of x-axis, y-axis and z-axis directions, respectively
i_1, i_2	rotation angles
L	rotation matrix
M	single scattering Mueller matrix
m	total number of nodes in the computational domain
N	nodal basis function
N_c	number of the incident directions for collimated radiation sources
N_{loc}	number of the local nodes in $V_{\mathbf{r}}$
p	vector of Legendre polynomial
p	order of Legendre polynomial
p_j	polynomial basis function of jth order
Q	linear polarization aligned parallel or perpendicular to the z-axis, W/m^2 sr
$\mathbf{R}_d$	reflection matrix for Lambertian surface
r	spatial coordinate vector
r	distance between $\mathbf{r_i}$ and $\mathbf{r}$
$\overline{\mathbf{S}}$	Stokes vector matrix
S	Stokes vector
U	linear polarization aligned $\pm 45^\circ$ to the z-axis, W/m^2 sr
V	circular polarization, W/m^2 sr
$V_{\mathbf{r}}$	spatial subdomain
w	weight function
Z	scattering phase matrix

Greek symbols

δ	Dirac delta function
φ	azimuth angle
$\boldsymbol{\kappa}_e$	extinction matrix
κ_e	extinction coefficient, m^{-1}
κ_s	scattering coefficient, m^{-1}
λ	support domain amplifying factor
μ, η, ξ	direction cosines of x-axis, y-axis and z-axis directions, respectively
Θ	scattering angle
θ	zenith angle
ρ	reflection coefficient
τ	optical thickness
$\mathbf{\Omega}$, $\mathbf{\Omega}'$	vector of radiation direction
Ω'	solid angle, sr
ω	single scattering albedo

Subscripts

0	inflow
c	collimated light
col	column
d	diffuse light
i	incident direction
loc	local nodes
row	row
s	scattered dirction
w	wall

Superscript

T	transposition

methods [16–19], successive order of scattering (SOS) methods [20–23], matrix operator methods [24], etc. To compare computational performance for different numerical methods, Kokhanovsky et al. [25] summarized seven codes of vector radiative transfer, including three techniques based on the DO method, two MC methods, the SOS method, and a modified doubling–adding technique. In that reference, benchmark data for one-dimensional vector atmospheric radiative transfer were presented.

Recently, a new class of meshless methods has been applied to solve the scalar radiative transfer equation (SRTE) in scattering media [26,27]. Compared to traditional numerical methods based on equation discretization (such as the finite difference method, the finite volume method and the finite element method), meshless methods have an advantage, since they only use scattered nodes to discretize the domain of the problem. In addition, the meshless method is much faster than ray tracing methods such as MC and zone methods. Due to the characteristics of flexibility and convenience, meshless methods have been developed to solve radiative transfer problems. To improve the numerical stability of meshless methods, a new second-order form of SRTE was presented by Zhao et al. [28], and Luo et al. [29] applied a kind of upwind scheme to solve the SRTE for strongly inhomogeneous media. To date, the meshless method hasn't been used to solve VRTE.

In this work, a direct collocation meshless method (DCM) is extended to solve the multidimensional VRTE. In this method, the angular discretization is based on the discrete-ordinates approach, where the spatial discretization is conducted using a collocation approach. Here, the trial function is constructed by a moving least squares approximation. Several test cases of polarized radiative transfer are taken to verify the performance of the method. The paper is organized as follows. Firstly, the theory of VRTE is introduced, and Lambertian boundary conditions are presented. Then, the formulation and solution steps of the DCM are presented in detail. In Section 3, two one-dimensional cases are studied to test the stability and convergence characteristics of the present method. Finally,