Journal of Quantitative Spectroscopy & Radiative Transfer 164 (2015) 97–108

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Coherent scattering by a collection of randomly located obstacles – An alternative integral equation formulation

Gerhard Kristensson

Department of Electrical and Information Technology, Lund University, P.O. Box 118, SE-221 00 Lund, Sweden

ARTICLE INFO

Article history:
Received 30 March 2015
Received in revised form
1 June 2015
Accepted 2 June 2015
Available online 11 June 2015

Keywords:
Electromagnetic scattering
Random scatterers
Coherent scattering
Null field approach
Integral equations

ABSTRACT

Scattering of electromagnetic waves by discrete, randomly distributed objects is addressed. In general, the non-intersecting scattering objects can be of arbitrary form, material and shape. The main aim of this paper is to calculate the coherent reflection and transmission characteristics of a slab containing discrete, randomly distributed scatterers. The integral representation of the solution of the deterministic problem constitutes the underlying framework of the stochastic problem. Conditional averaging and the employment of the Quasi Crystalline Approximation lead to a system of integral equations in the unknown expansion coefficients. Of special interest is the slab geometry, which implies a system of integral equations in the depth variable. Explicit solutions for tenuous media and low frequency approximations can be obtained for spherical obstacles.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

Multiple scattering of electromagnetic waves by a discrete collection of scatterers has a long and well-documented history in the literature. Typical applications of the results are found at a wide range of frequencies (radar up to optics), such as attenuation of electromagnetic propagation in rain, fog, and clouds *etc*. The topic is covered in detail in several textbooks [1–7] and the interested reader is referred to these excellent treatments for discussions on the subject.

The journal literature is also extensive. The starting point of multiple scattering can be set to the pioneer work by Foldy [8]. Several important works followed [9–26] and further references to the subject are found therein. Some of the theories are tested experimentally [22,27,28].

The main stress in the cited literature above is on finding the effective electromagnetic properties, *i.e.*, the bulk permittivity and the permeability of the many-scatterer system. This is effectively done by the introduction of an exponential trial solution in the final equations, which leads to a determinant system in the unknowns. The field inside and outside the scattering region is then computed. One purpose of this paper is to develop an alternative, computationally effective method, that does not rely on the homogenized properties of the problem, but has a potential to be useful at frequencies outside the frequency region of homogenization.

The analysis presented in this paper shows initial similarities with previous treatments of the topic [25], but then proceeds along a somewhat different route, and the transmission and reflection properties of a slab geometry are in focus. The details of the results presented in this paper can be found in the underlying technical report [29]. The transmitted and reflected intensities are conveniently represented as a sum of two terms – the coherent and the incoherent contributions. In this paper we focus on the analysis of the coherent term. The remaining part – the incoherent or diffuse part – is postponed to a future paper.

A system of integral equations is identified, and the reflected and transmitted (coherent) fields are identified. Integral equations, especially if the kernel is smooth, have well established properties and are easy to solve. This is a major advantage of the approach presented in this paper.

If desired, the transmitted field may then be used to find the effective bulk material parameters, and, consequently, the

E-mail address: Gerhard.Kristensson@eit.lth.se

http://dx.doi.org/10.1016/j.jqsrt.2015.06.004
0022-4073/© 2015 Elsevier Ltd. All rights reserved.

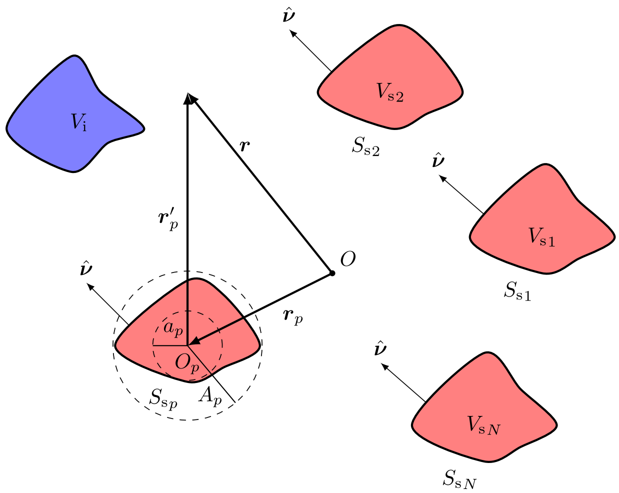

Fig. 1. The geometry of a collection of the N scatterers and the region of prescribed sources $V_{\rm i}$. The positions of the local origins are $\boldsymbol{r}_p$, $p=1,\dots,N$, and the radii of the maximum inscribed and the minimum circumscribed spheres of each local scatterer are a_p and A_p, respectively.

field inside the material. This procedure also introduces a means of estimating the accuracy of the bulk property approximation by comparing the accuracy of the transmitted and reflected field with the corresponding results obtained by a homogeneous slab. If the bulk material parameters model the material appropriately, both transmission and reflection fields computed with the two methods agree. This procedure is illustrated numerically in a subsequent paper [30].

The transmitted coherent contribution is frequently modeled by the use of the Radiative Transfer Equation RTE) – more precisely the Bouguer–Beer law [1,7,31]. The RTE is derived under certain assumptions, *e.g.*, the scatterers are far enough apart, so that they are located in the far field of all other scatterers, and plane wave excitation of the scatterers (single scatterer extinction cross section is used). These assumptions are not made in this paper. In this respect, the present analysis generalizes this law.

The paper is organized as follows. In Section 2, a review of the deterministic analysis of the multiple electromagnetic scattering is given and solved with the use of spherical vector wave and their translation properties. This analysis follows to a large extent the pioneer work made by Peterson and Ström [15] in the early 1970s. The final expressions, however, differ, due to different aim of the analysis. The stochastic description is made in Section 3. Conditional averaging and the employment of the Quasi Crystalline Approximation lead to a system of integral equations in the unknown expansion coefficients. The details of this analysis is given in Section 4. The pertinent system of integral equations for the slab (and the half space) is developed in Section 5, and two natural and important approximations – tenuous media and low-frequency approximations – are developed in Section 6. The paper ends with a short conclusion in Section 7 and an appendix.

2. The null-field approach to a collection of scatterers

We study a collection of N different scatterers, where each scatterer is centered at the location $\boldsymbol{r}_p$, defining the position of the local origin O_p, $p=1,2,\dots,N$, relative the global origin O, see Fig. 1. The radii of the maximum inscribed and the minimum circumscribed spheres, both centered at the local origin, of each scatterer are denoted a_p and A_p, $p=1,2,\dots,N$, respectively. The scatterers are located in a lossless, homogeneous, isotropic media with permittivity ϵ and permeability μ. The wave number and relative wave impedance are k and η, respectively – both real numbers. We assume that no circumscribing spheres intersect. Each scatterer has its own material properties, which do not have to be the same for all scatterers. The prescribed sources are located in the region $V_{\rm i}$, which is a region disjoint to all scatterers, and these sources generate the field $\boldsymbol{E}_{\rm i}(\boldsymbol{r})$ everywhere outside $V_{\rm i}$. More precisely, the circumscribing sphere of the source region must not include any local origin $\boldsymbol{r}_p$, $p=1,2,\dots,N$. Of course, an incident plane wave fulfil these restrictions.

The solution to the deterministic many-body scattering problem of electromagnetic waves by means of the null-field approach (T-matrix approach or Waterman's method in honor of Peter Waterman [32]) has been reported several times in the literature. An early attempt goes back to Peterson and Ström [15]. Several recent treatment of the problem, *e.g.*, [5,25], makes it unnecessary to go through the details in this paper. The method is well-documented, see the comprehensive database [33–37]. The reader interested in the explicit details and the notation used in this paper is referred to [29]. Below, we therefore only give a short review of the pertinent equation of the analysis and the essential equations used below.

The total electric field is decomposed in the incident and the scattered field $\boldsymbol{E}=\boldsymbol{E}_{\rm i}(\boldsymbol{r})+\boldsymbol{E}_{\rm s}(\boldsymbol{r})$. The incident field is specialized to a plane wave impinging along the direction $\hat{\boldsymbol{k}}_{\rm i}$ with an expansion in regular spherical vector waves, $\boldsymbol{v}_n(k\boldsymbol{r})$ [38], *i.e.*,

$$\boldsymbol{E}_{\rm i}(\boldsymbol{r})=\boldsymbol{E}_0\mathrm{e}^{\mathrm{i}k\hat{\boldsymbol{k}}_{\rm i}\cdot\boldsymbol{r}}=\sum_n a_n\boldsymbol{v}_n(k\boldsymbol{r}) \tag{1}$$

where the index n is a multi-index,[1] and where $\boldsymbol{E}_0$ is the field value of the plane wave at the common origin O. The expansion coefficients, a_n, are given in terms of the vector

[1] Depending on the context, the index n consists of three or four different indices, *i.e.*, $n=\sigma ml$ or $n=\tau\sigma ml$, where $\tau=1,2$, $\sigma=\mathrm{e},\mathrm{o}$, $m=0,1,2,\dots,l$, and $l=1,2,3,\dots$. Both conventions are used in this paper.

Download English Version:

https://daneshyari.com/en/article/5427981

Download Persian Version:

https://daneshyari.com/article/5427981

Daneshyari.com