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Polarized bidirectional reflectance of optically thick sparse particulate layers: An efficient numerically exact radiative-transfer solution

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ABSTRACT

We describe a simple yet efficient numerical algorithm for computing polarized bidirectional reflectance of an optically thick (semi-infinite), macroscopically flat layer composed of statistically isotropic and mirror symmetric random particles. The spatial distribution of the particles is assumed to be sparse, random, and statistically uniform. The 4×4 Stokes reflection matrix is calculated by iterating the Ambartsumian's vector nonlinear integral equation. The result is a numerically exact solution of the vector radiative transfer equation and as such fully satisfies the energy conservation law and the fundamental reciprocity relation. Since this technique bypasses the computation of the internal radiation field, it is very fast and highly accurate. The FORTRAN implementation of the technique is publicly available on the World Wide Web at http://www.giss.nasa.gov/staff/mmishchenko/brf. It can be combined with several existing computer programs providing the requisite single-scattering properties of spherical or morphologically complex particles and applied to a wide range of optical characterization problems. Benchmark results obtained with this program can be used for testing alternative solvers of the vector radiative transfer equation.

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1. Introduction

In a 1999 paper [1], we described an efficient numerical solution of the Ambartsumian's nonlinear integral equation satisfied by the reflection function of an optically semi-infinite, homogeneous layer of sparse discrete random medium. The corresponding FORTRAN implementation of this solution [2] has been used extensively by the research community to compute the bidirectional reflection function of optically thick particulate layers. The obvious limitation of the computer program [2] is its reliance on the scalar approximation [3] and the resulting inability to compute the polarized bidirectional reflection function. Therefore, the main objective of this paper is to describe in detail a natural extension of Ref. [1] wherein the scalar nonlinear integral equation is replaced by its full vector version as well as to serve as a step-by-step user guide to the corresponding computer programs. While the amount of new science in this user guide (similar in style to Refs. [4–11]) is by definition limited, the development of

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a numerical algorithm that is both straightforward, user-friendly, and robust is always non-trivial and deserves a thorough discussion. We expect that the resulting computer programs [2] will be used both as an efficient analysis tool and as a source of benchmark numerical results suitable for testing alternative computer solvers of the vector radiative transfer equation.

To make the discussion in this paper more compact, we will use throughout the terminology and notation adopted in the monographs [3,12,13]. Note that Refs. [3,12] are available on-line as PDF files at http://www.giss.nasa.gov/staff/mmishchenko/books.html as well as at https://www.researchgate.net/profile/Michael_Mishchenko.

2. Vector nonlinear integral equation

Let us consider a semi-infinite, plane-parallel, statistically uniform layer of sparse discrete random medium extending in the vertical direction from $z=0$ to $z=-\infty$, where the z-axis of the laboratory right-handed coordinate system is perpendicular to the boundary of the medium and is directed upwards (Fig. 1). This implies that the particles constituting the layer are imbedded only in the lower half of the infinite homogeneous host medium. The host medium is assumed to be nonabsorbing.

A propagation direction $\hat{\mathbf{n}}$ at a point in space will be specified by a couplet $\{u, \varphi\}$, where $u = -\cos\theta \in [-1, +1]$ is the direction cosine, while θ and φ are the corresponding polar and azimuth angles with respect to the local coordinate system having the same spatial orientation as the laboratory coordinate system (Fig. 1). As usual, the polar (zenith) angle $\theta \in [0, \pi]$ is measured from the positive z-axis and the azimuth angle $\varphi \in [0, 2\pi)$ is measured from the positive x-axis in the clockwise direction when looking in the direction of the positive z-axis. A positive u always corresponds to a

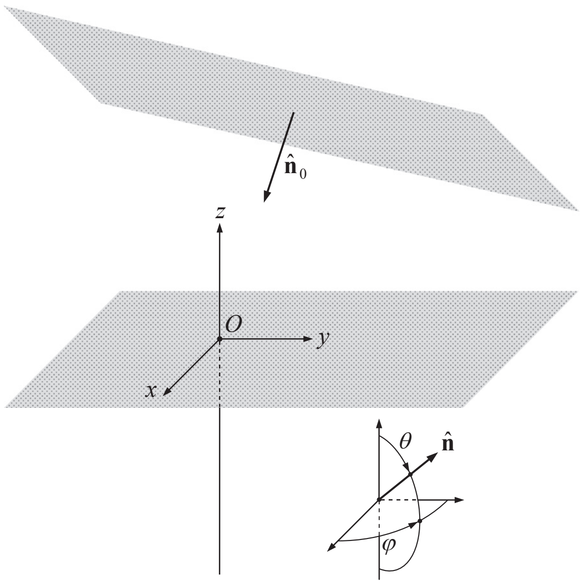

Fig. 1. Spherical coordinates used to specify the direction of light propagation.

downward direction, whereas a negative u always corresponds to an upward direction. It is also convenient to introduce a nonnegative quantity $\mu = |u| \in [0, 1]$.

The random particulate layer is illuminated from above by a plane electromagnetic wave or a parallel quasi-monochromatic beam of light propagating in the direction $\hat{\mathbf{n}}_0 = \{u_0, \varphi_0\} = \{\mu_0, \varphi_0\}$. The uniformity and infinite transverse extent of the wave or the beam combined with the statistical uniformity of the particulate layer ensure that all parameters of the diffuse radiation field are independent of the coordinates x and y.

We assume the $\exp(-i\omega t)$ time-harmonic dependence of the electromagnetic field, where t is the time, ω is the angular frequency, and $i = (-1)^{1/2}$. This assumption implies a non-negative imaginary part of the particle relative refractive index. In accordance with Refs. [3,12,13], the four-component Stokes column vector of the incident light is defined as

$$\mathbf{I}_0 = \begin{bmatrix} I_0 \\ Q_0 \\ U_0 \\ V_0 \end{bmatrix} = \frac{1}{2}\sqrt{\frac{\varepsilon_1}{\mu_0}} \begin{bmatrix} \langle\langle E_{0\theta}E_{0\theta}^* + E_{0\varphi}E_{0\varphi}^* \rangle\rangle \\ \langle\langle E_{0\theta}E_{0\theta}^* - E_{0\varphi}E_{0\varphi}^* \rangle\rangle \\ \langle\langle -E_{0\theta}E_{0\varphi}^* - E_{0\varphi}E_{0\theta}^* \rangle\rangle \\ i\langle\langle E_{0\varphi}E_{0\theta}^* - E_{0\theta}E_{0\varphi}^* \rangle\rangle \end{bmatrix}, \tag{1}$$

where ε_1 is the real-valued electric permittivity of the infinite host medium; μ_0 is the permeability of a vacuum (not to be confused with the direction cosine $\mu_0 = u_0$); $\langle\langle\cdots\rangle\rangle$ denotes averaging over a "sufficiently long" period of time; $E_{0\theta}$ and $E_{0\varphi}$ are the θ- and φ-components of the electric field vector, respectively; and the asterisk denotes a complex-conjugate value. Note that the longitudinal component of the incident electric field is equal to zero since the electromagnetic field of the incident light is transverse. The same conventions as in Eq. (1) are used to define the 4-component diffuse specific intensity column vector $\tilde{\mathbf{I}}_d(z, \hat{\mathbf{n}}) = \tilde{\mathbf{I}}_d(z, u, \varphi)$. The dimension of $\mathbf{I}_0$ is W m^{-2}, while that of $\tilde{\mathbf{I}}_d$ is W m^{-2} sr^{-1}.

We assume that the random particles forming the scattering layer are statistically isotropic and mirror symmetric [3,13]. This allows one to fully characterize the optical properties of the particulate layer by the ensemble-averaged single-scattering albedo ϖ and so-called normalized 4×4 Stokes scattering matrix $\tilde{\mathbf{F}}(\Theta)$ with real-valued components. The latter has the well-known block-diagonal structure:

$$\tilde{\mathbf{F}}(\Theta) = \begin{bmatrix} a_1(\Theta) & b_1(\Theta) & 0 & 0 \\ b_1(\Theta) & a_2(\Theta) & 0 & 0 \\ 0 & 0 & a_3(\Theta) & b_2(\Theta) \\ 0 & 0 & -b_2(\Theta) & a_4(\Theta) \end{bmatrix}, \tag{2}$$

where $\Theta \in [0, \pi]$ is the angle between the incidence and scattering directions (i.e., the scattering angle). The (1,1) element (often called the phase function) is non-negative and satisfies the normalization condition:

$$\frac{1}{2}\int_0^{\pi} d\Theta \, \sin\Theta a_1(\Theta) = 1. \tag{3}$$

The assumption of low packing density of the particles forming the scattering slab allows one to compute the diffuse specific intensity column vector $\tilde{\mathbf{I}}_d(z, \hat{\mathbf{n}})$ by solving