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Characterization of supersonic radiation diffusion waves

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ABSTRACT

Supersonic and diffusive radiation flow is an important test problem for the radiative transfer models used in radiation-hydrodynamics computer codes owing to solutions being accessible via analytic and numeric methods. We present experimental results with which we compare these solutions by studying supersonic and diffusive flow in the laboratory. We present results of higher-accuracy experiments than previously possible studying radiation flow through up to 7 high-temperature mean free paths of low-density, chlorine-doped polystyrene foam and silicon dioxide aerogel contained by an Au tube. Measurements of the heat front position and absolute measurements of the x-ray emission arrival at the end of the tube are used to test numerical and analytical models. We find excellent absolute agreement with simulations provided that the opacity and the equation of state are adjusted within expected uncertainties; analytical models provide a good phenomenological match to measurements but are not in quantitative agreement due to their limited scope.

1. Introduction

Phenomena in which the flux of radiation emitted by a heated body ($\mathbf{S}_r = \sigma T^4$) exceeds the conduction of energy by heated material ($\varepsilon \rho C_s$) occur in a range of exotic plasmas from the laboratory to astrophysical scale. Here σ is the Stefan–Boltzmann constant (W/m^2/K^4), T is the temperature (K), ε is the specific internal energy (J/g), ρ is the mass density (g/m^3), and C_s is the sound speed (m/s). In 3D this is a difficult and complex class of physical problem in which not only is the equation of radiative transfer a function of seven independent variables. While in a simpler 1D planar geometry this reduces to four variables, account must still be taken of the complex dependence of material properties (opacity and ε) on the relevant independent variables. Through various approximations analytical solutions to 1D problems have been found, but owing to the complexity of almost all 2D and 3D problems, solutions are necessarily numerical via computer simulation. In many cases these computer codes are also used to simulate and answer questions about the structure of stars, the behaviour of supernovae and closer to home, in the laboratory, the transport of x-ray energy in

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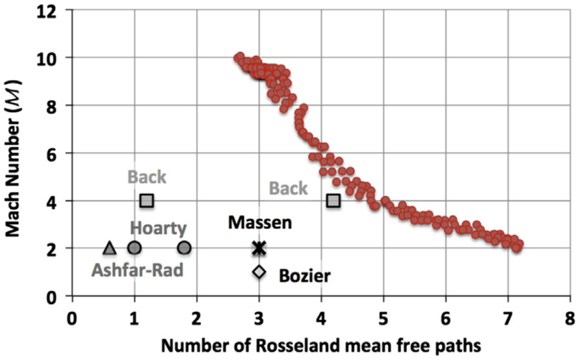

Fig. 1. Radiation flow Mach number $\mathcal{M}$ compared to the number of Rosseland mean free paths at the material temperature; calculations of the Rosseland mean free path vs time for the experiments we present are extracted directly from simulations and are shown by the red points. Previous experiments include [10–14]. (For interpretation of the references to colour in this figure caption, the reader is referred to the web version of this paper.)

Inertial Confinement Fusion targets [1–3]. As with all simulations it is essential that they be benchmarked against data and analytic reference problems to verify the numerical procedures.

In this paper we detail the first experimental measurements in a previously inaccessible regime to examine one of the few semi-analytic solutions that does exist in this area – the 'Marshak wave' problem [4–8]. As one of the few problems dealing with radiation transport that has analytic solutions it is often used to provide quantitative validation of numerical results. We generate experimental results to test the applicability of several previously published analytic approximations and fully integrated radiation hydrocode simulations.

We present results that characterize supersonic and diffusive radiation flow in a highly diffusive regime, using a 310 eV temperature x-ray source to drive a radiation heat front at downstream Mach ($\mathcal{M}=6$) through more than 6 Rosseland mean free paths. The key to reaching such regimes lies with the laser energy available on the National Ignition Facility (NIF) [9]. Results from previous experiments are summarized in Fig. 1. These were typically limited to regimes with velocities of $\mathcal{M}=2$–4 and investigated flow of a lower radiation temperature (120–150 eV) through 3–4 mean free paths [10,11,13,12]. In this regime the ionization of material does not lead to an energetically significant re-radiated flux, and thus the experiments were not in a fully diffusive regime. Experiments by [14] extended the radiation drive to $\sim$190 eV, but were limited by the foam-scale and laser energy [14]. The energy now available on the NIF enables a more constraining class of experiment using higher, more uniform density foam that is more accurately characterized, and generating a brighter heat front from which more accurate measurements of x-ray emission are made than was previously possible.

In Section 2 we describe the theoretical problem, develop the arguments behind the simple diffusion model from which much of the analytical results described in Section 2.1 are developed. In Section 2.2 we describe the different numerical approximations that we later apply to compare with the experimental results. The experiment is detailed in Section 3.1, with the primary results described in Sections 3.2 and 3.3. Finally, the comparison of measurements to the various analytical and numerical solutions is discussed in Section 4.

2. The general problem and solutions

The 1D problem is typically posed as a cold absorbing slab medium occupying a semi-infinite half-space [6,15]. The medium is assumed to be perfectly homogeneous and initially at zero temperature with no external radiation sources present. A constant temperature radiation source is applied at the boundary, heating the surface and reducing its opacity. Solutions aim to find the position of the heat-front vs time as it penetrates into the cold medium and the spatial shape of the temperature profile:

$$\frac{\partial}{\partial t}\left[\rho\varepsilon+\frac{\rho u^2}{2}+U_r\right] = -\nabla\cdot\left[\rho\mathbf{u}\left(\varepsilon+\frac{u^2}{2}\right)+(P_m+P_r)\mathbf{u}+\mathbf{S}_r\right] \quad (1)$$

Beginning with the energy equation containing external radiation sources – Eq. (1) – several simplifying assumptions are made immediately (1) that radiation energy density (U_r) and pressure (P_r) are negligible compared to material internal and kinetic energy and pressure (P_m), and (2) that in the supersonic limit any material motion (u) can be ignored. In Eq. (1) ρ is the mass density, u the velocity, ε the material internal energy, and $\mathbf{S}_r$ the radiant energy flux.

In addition to the energy equation we must consider the kinetic equation for the distribution function of photons which describes how radiation energy is transferred into the material. If scattering is ignored, then in the conventional form the equation of transfer of the specific radiation intensity $I(\nu,\boldsymbol{\Omega},\mathbf{r},t)$ can be written:

$$\frac{1}{c}\frac{\partial I(\nu)}{\partial t}+\boldsymbol{\Omega}\cdot\nabla I(\nu)=\kappa'(\nu)\left(I_p(\nu)-I(\nu)\right) \quad (2)$$

where κ' is the frequency dependent opacity modified for induced emission according to Kirchoff's law, $I_p(\nu)$ is Plank's function for the equilibrium spectral energy density from a blackbody as a function of frequency (ν) [16], and c is the speed of light in vacuum.

Without going into the details of the underlying assumptions of the diffusion approximation which are well-described by Pomraning and Castor, the basic argument relies on the assumption that the angular dependence of the specific radiation intensity is only very weakly anisotropic and can be represented by the first two terms in a spherical expansion i.e. $I(\nu,\boldsymbol{\Omega})=I_0(\nu)/4\pi+3\boldsymbol{\Omega}\dot{\mathbf{I}}_1(\nu)/4\pi$ [17,18]. Pomraning elegantly describes how the angular moment of Eq. (2) reduces to a Fick's law form if the radiation field be treated as quasi-steady-state by neglecting the time-derivative term [19,20]. In this approximation it is simple to identify that $I_0(\nu)$ is $cU_r(\nu)$ and $\mathbf{I}_1(\nu)$ is $\mathbf{S}_r(\nu)$ yielding the relationship between the flux and radiation energy density in the diffusion limit:

$$\mathbf{S}_r=-\frac{c}{3}\rho\kappa'\nabla U_r \quad (3)$$