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Spatial and angular finite element method for radiative transfer in participating media

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ABSTRACT

A computational approach for the modeling of multi-dimensional radiative transfer in participating media, including scattering, is presented. The approach is based on the sequential use of angular and spatial Finite Element Methods for the discretization of the Radiative Transfer Equation (RTE). The angular discretization is developed with an Angular Finite Element Method (AFEM) based on the Galerkin approach. The AFEM leads to a counterpart of the RTE consisting of a coupled set of transient-advective–reactive equations that are continuously dependent on space and time. The AFEM is ideally suited for so-called *h*- and/or *p*-refinement for the discretization of the angular domain: *h*-refinement is obtained by increasing the number of angular elements and *p*-refinement by increasing the order of the angular interpolating functions. The spatial discretization of the system of equations obtained after the angular discretization is based on a Variational Multi-Scale Finite Element Method (VMS-FEM) suitable for the solution of generic transport problems. The angularly and spatially discretized system is solved with a second-order accurate implicit predictor multi-corrector time stepper together with a globalized inexact Newton–Krylov nonlinear solver. The overall approach is designed and implemented to allow the seamless inclusion of other governing equations necessary to solve coupled fluid–radiative systems, such as those in combustion, high-temperature chemically reactive, and plasma flow models. The combined AFEM and VMS-FEM for the solution of the RTE is validated with two- and three-dimensional benchmark problems, each solved for 3 levels of angular partitioning (*h*-refinement) and for 2 orders of angular basis functions (*p*-refinement), i.e. piecewise constant (P_0) and piecewise linear (P_1) basis over spherical triangles. The overall approach is also applied to the simulation of radiative transfer in a parabolic concentrator with participating media, as encountered in solar thermochemical applications, for different values of absorption and scattering coefficients, and for different angles of inclination of the incident radiation.

1. Introduction

1.1. Radiative transfer in participating media

Radiative transfer in participating media takes place in a wide range of natural phenomena and engineering applications. Radiative transfer refers to the transport of particles, including photons in electromagnetic waves [1], without the requirement of an underlying medium of propagation. When radiative transfer takes place in radiatively participating media, the radiative intensity can be attenuated and/or augmented depending on the characteristics of the medium. Some examples of radiative transfer interacting with media in natural phenomena are the interaction of solar radiation with air or carbon dioxide in the earth's atmosphere, and the interaction of

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Nomenclature

A	area of an spherical triangle
$\mathbf{A}_0$	transient transport matrix of the VMS-FEM
$\mathbf{A}_i$	advective transport matrix for spatial direction i of the VMS-FEM
c	Speed of light in the medium
d	subscript for the numbering of a discrete direction
$\mathbf{F}$	scattering matrix of the AFEM discretization of the RTE
G	total incident radiation
i	spatial coordinate index, e.g. for three-dimensional Cartesian coordinates $i=\{x, y, z\}$
$I=I(t,\mathbf{x},\mathbf{s})$	total radiative intensity at time t, spatial location $\mathbf{x}$, and along direction $\mathbf{s}$
$I_b=I_b(T)$	total black body intensity
I_h	radiative intensity approximated by the angular basis function $\mathbf{N}^s$
I_0	initial condition for the radiative field I in the RTE
$\mathbf{I}$	column vector with angularly discretized values of total intensity
$\mathbf{I}_h$	vector of angularly and spatially discretized radiative intensity
k_B	Boltzmann's constant
$\mathbf{K}_{ij}^{DC}$	discontinuity-capturing diffusivity matrix for the VMS-FEM
l	discretization level
$\mathcal{L}r$	differential–integral operator characterizing the RTE
$\mathcal{L}$	differential operator characterizing the angularly discretized RTE
$\mathbf{L}$	interpolation vector of the AFEM discretization of the RTE
$\mathbf{M}$	mass matrix of the AFEM discretization of the RTE
n	refractive index
n_d	number of discrete directions
n_e	number of angular elements discretizing S
n_v	number of angular element vertices
$\mathbf{n}$	normal vector, positive towards the inside of the spatial domain Ω
$\mathbf{n}_w$	surface normal vector to a wall
$\mathbf{N}^s=\mathbf{N}^s(\mathbf{s})$	set of angular basis functions
$\mathbf{N}^x=\mathbf{N}^x(\mathbf{x})$	set of spatial basis functions
p	Dirichlet (i.e. specified value) boundary condition for I
$\mathbf{q}_r$	radiative heat flux
$\mathbf{q}_{r,inc}$	incident radiative heat flux
$\mathbf{Q}_i$	radiative heat flux coefficient matrix for direction i of the AFEM
$\dot{Q}$	volumetric heat generation term
$\mathbf{x}$	vector of spatial coordinates, e.g., $\mathbf{x}=[x, y, z]$ for three-dimensional space in Cartesian coordinates
$\mathscr{R}=\mathscr{R}(I)$	residual form of the RTE
$\mathscr{R}^s=\mathscr{R}^s(\mathbf{I})$	residual form of the angularly discretized RTE
$\mathbf{R}=\mathbf{R}(\mathbf{I}_h)$	global residual vector of the angularly and spatially discrete RTE
S	unit sphere, representing the 4π solid angle
S_e	angular domain for element e over S; $S=\bigcup_e S_e$
$\mathbf{s}$	generic unit direction vector ($\lVert\mathbf{s}\rVert=1$) covering the surface of S
$\mathbf{s}\cdot\nabla=s_i\partial_i$	operator to calculate the rate of change in the direction of propagation
$\mathbf{S}_i$	directional matrix for the spatial direction i of the AFEM
$\mathbf{S}_0$	source vector of the VMS-FEM
$\mathbf{S}_1$	reactive transport matrix of the VMS-FEM
t	time
T	temperature
V	vertex defining an angular finite element over the unit sphere S
Γ	spatial domain boundary
Γ_h	discrete spatial domain boundary
ε	total hemispherical emittance of a surface; $\varepsilon=1$ for a black surface
κ	total absorption coefficient
ρ	total reflectance of a diffuse surface; $\rho=1-\alpha$ for opaque walls
ρ^s	specular surface reflectance
ρ^d	diffuse surface reflectance
σ	total scattering coefficient
σ_{SB}	Stefan–Boltzmann constant
τ	intrinsic time scales matrix of the VMS-FEM
$\Phi=\Phi(\mathbf{s}',\mathbf{s})$	scattering phase function dependent on directions $\mathbf{s}'$ and $\mathbf{s}$
ω, ω'	solid angles corresponding to directions $\mathbf{s}$ and $\mathbf{s}'$, respectively
Ω	spatial domain
Ω_h	discretized spatial domain

light from a star with interstellar media before reaching an observer on earth. Radiative transfer in participating media also takes place, and even plays a dominant role, in diverse engineering applications and industrial processes, such as combustion in rocket nozzles or in internal combustion engines, solar thermochemical synthesis, electrical discharges and lasers, and subatomic particle transport in nuclear reactors [2,3].

The present article focuses on radiative heat transfer, also called thermal radiation, (hereafter simply referred to as radiation), which corresponds to heat transfer caused by photons emitted by matter with temperature greater than absolute zero.

The equation used in most radiative transfer models is the Radiative Transfer Equation (RTE), which describes the balance of radiative energy as it propagates through a medium. Some approaches to solve this equation are described next.

1.2. Approaches for the solution of the RTE

Approaches for the solution of the RTE can be broadly divided between statistical (non-deterministic)