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Radiative heat flux predictions in hyperbolic metamaterials

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ABSTRACT

The transport of heat mediated by thermal photons in hyperbolic multilayer metamaterials is studied using the fluctuational electrodynamics theory. We discuss the dependence of the attenuation length and the heat flux on the design parameters of the multilayer structure. We demonstrate that in comparison to bulk materials the flux inside layered hyperbolic materials can be transported at much longer distances, making these media very promising for thermal management and for near-field energy harvesting.

1. Introduction

Nanoscale radiative heat transfer has attracted a lot of attention in the last few years because of Polder and van Hove's prediction [1] on the possibility to observe heat fluxes at subwavelength distances which are several orders of magnitude larger than those obtained by the blackbody theory. Recent experimental results [2–9] have confirmed these theoretical predictions [1,10,11].

This increased radiative heat transfer in the near-field regime might be used for different applications as for example near-field imaging [12–15], nanoscale thermal management by heat flux rectification, amplification and storage [16–23], and near-field thermophotovoltaics [24–29]. In particular, for near-field thermophotovoltaic (nTPV) applications it is desirable to have large heat fluxes which are quasi-monochromatic at the bandgap frequency of the thermophotovoltaic cell. Now, it could be shown theoretically that for phonon-polaritonic materials the heat flux is quasi-monochromatic at the surface phonon-frequency resulting in heat fluxes which can be orders of magnitude larger than the blackbody result [30] due to the large number of contributing surface modes [31]. This is the reason why phonon-polaritonic media are used in most experiments [3–6,8]. However, it should be kept in mind that there are also upper limits for this surface mode contribution as shown in [32–35].

On the other hand, the nanoscale heat flux between two halfspaces separated by a distance d which is due to surface modes is absorbed on a very thin layer of about $0.2d$ [36,37]. That means that when constructing for example a near-field thermophotovoltaic device choosing $d = 100$ nm most energy is already absorbed in a thin surface layer of about 20 nm. This is very unfavorable for applications in near-field thermophotovoltaic devices, since only the electron–hole pairs in this thin layer can effectively be used for energy conversion [27].

As could be shown recently [38,39] for the so-called hyperbolic or indefinite materials [40], which exist naturally [41–43] but can also be constructed artificially by combining

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layers of a dielectric and a plasmonic/polaritonic material [44–46] or by using plasmonic/polaritonic nanowire structures [40,47–50,41,51], the nanoscale heat radiation by hyperbolic modes can result in heat fluxes which are on the order of or even larger than the heat flux by surface modes [38]. This is due to a broad band of hyperbolic modes which are, in fact, frustrated total internal reflection modes. Recently, hyperbolic structures were proposed for applications in nTPV [52,53].

Having a broad frequency band for nanoscale heat radiation seems to be disadvantageous for nTPV, but this disadvantage is compensated by a striking property of hyperbolic modes: hyperbolic modes are propagating modes inside the hyperbolic metamaterials and therefore it can be expected that the penetration depth is much larger than for surface modes. Hence, the effective layer on which electron–hole pairs are generated can be orders of magnitude larger than for surface-mode driven heat transfer. This property could be shown by means of an effective description, presented very recently by us [54]. But such effective descriptions should be taken with care for describing near-field thermal radiation, since it tends to overestimate the hyperbolic contribution to the heat flux [55,56] and it does not correctly describe the surface modes of the composite materials of the hyperbolic structure [56,57].

In this paper, we study the penetration depth of the energy flow in multilayer hyperbolic materials using an exact S-matrix method [58,59] based on Green's function [60] formalism combined with fluctuational electrodynamics [61]. We have previously shown that the attenuation length can be very large for hyperbolic nanowire and multilayer materials using an effective medium description [54]. Here, we use the exact formalism to study the energy flux and penetration depth for multilayer hyperbolic metamaterials. The exact formalism is compared with effective medium theory to better understand potential limitations of the latter. We emphasize that the here developed method can directly be used to make exact calculation for the energy streamlines inside hyperbolic multilayer structures which were treated only within the effective medium approach so far [62].

2. Energy flux inside a layered medium

The theoretical description of near-field heat radiation is in most studies based on fluctuational electrodynamics [61]. Within this theory it is assumed that the thermal fluctuating fields of a dielectric body which is assumed to be in local thermal equilibrium at a temperature T are on a macroscopic scale due to fluctuational source current densities. Hence, Maxwell's equations are augmented by fluctuational Gaussian source currents $\mathbf{J}^{\mathrm{m}}$ and $\mathbf{J}^{\mathrm{e}}$ yielding

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\mathbf{J}^{\mathrm{m}}(\mathbf{r},t) - \frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}, \tag{1}$$

$$\nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J}^{\mathrm{e}}(\mathbf{r},t) + \frac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}. \tag{2}$$

For nonmagnetic materials the fluctuating magnetic source currents can be neglected: $\mathbf{J}^{\mathrm{m}}(\mathbf{r},t) = \mathbf{0}$. The source current density $\mathbf{J}^{\mathrm{e}}(\mathbf{r},t)$ is assumed to have zero mean value: $\langle \mathbf{J}^{\mathrm{e}} \rangle = \mathbf{0}$, where the brackets symbolize the ensemble average. Then it is further assumed that the second moment or correlation function of the source currents is given by the fluctuation dissipation theorem of second kind [63]:

$$\langle J^{\mathrm{e}}_{\alpha}(\mathbf{r},\omega) J^{\mathrm{e}}_{\beta}(\mathbf{r}',\omega') \rangle = 4\pi\omega \Theta(\omega,T)\epsilon_{\mathrm{vac}}\epsilon''_{\alpha\beta}\delta(\mathbf{r}-\mathbf{r}')\delta(\omega+\omega'), \tag{3}$$

where $\Theta(\omega,T) = \hbar\omega/(\mathrm{e}^{\hbar\omega/k_{\mathrm{B}}T}-1)$ and $\epsilon''_{\alpha\beta}$ is the imaginary part of the permittivity tensor of the considered material; ϵ_{vac} is the permittivity of vacuum, $2\pi\hbar$ is Planck's constant, k_{B} is Boltzmann's constant, ω is the circular frequency, and δ stands for the delta function. Here, obviously quantum mechanics in form of the fluctuation dissipation theorem enters into the theoretical description which can therefore be regarded as a semi-classical theory. However, a full quantum mechanical description agrees with this method [64].

Now, since the fields are linearly related to the sources they can be expressed as

$$\mathbf{E}(\mathbf{r},\omega) = \mathrm{i}\omega\mu_{\mathrm{vac}} \int_V \mathrm{d}^3\mathbf{r}'\ \mathbb{G}^{\mathrm{E}}(\mathbf{r},\mathbf{r}';\omega)\cdot \mathbf{J}^{\mathrm{e}}(\mathbf{r}',\omega), \tag{4}$$

$$\mathbf{H}(\mathbf{r},\omega) = \mathrm{i}\omega\mu_{\mathrm{vac}} \int_V \mathrm{d}^3\mathbf{r}'\ \mathbb{G}^{\mathrm{H}}(\mathbf{r},\mathbf{r}';\omega)\cdot \mathbf{J}^{\mathrm{e}}(\mathbf{r}',\omega) \tag{5}$$

by introducing the dyadic Green's functions $\mathbb{G}^{\mathrm{E}}$ and $\mathbb{G}^{\mathrm{H}}$. Since we only consider nonmagnetic materials μ_{vac} is the permeability of vacuum and of all materials. By means of the fluctuation dissipation theorem we can now derive the mean Poynting vector or Maxwell's stress tensor, for instance. For some general elaborations on the stress tensor and the Poynting vector within the formalism of fluctuational electrodynamics we refer the interested reader to [65]. Since we are interested in heat radiation we focus on the Poynting vector.

Let us now assume that we have a situation as depicted in Fig. 1. For $z < z_0 = 0$ we have a semi-infinite isotropic material which is at local thermal equilibrium at temperature T_0. This halfspace is separated by a vacuum gap of size d from a second halfspace which can be any kind of multilayer structure and which is assumed, for sake of clarity, to be at zero temperature. This assumption means that this medium does not emit thermal photons but it can only scatter and absorb them. However, of course, this medium could be set at any temperature. Straightforwardly we obtain the expression (using the Einstein convention):

$$\langle S_z \rangle = 2\mathrm{Re} \int_0^\infty \mathrm{d}\omega\ \Theta(\omega,T_0) \frac{\mu_{\mathrm{vac}}^2 \omega^3\ \mathrm{Im}(\epsilon_0)}{\pi}$$

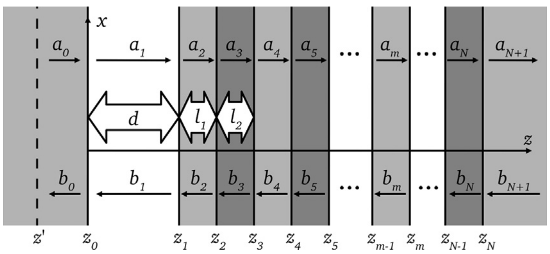

Fig. 1. Sketch of the considered geometry: for $z < z_0$ halfspace filled with GaN; for $z_0 < z < z_1$ vacuum gap of width d; for $z_1 < z < z_N$ bilayer structure with a period $\Lambda = l_1 + l_2$. The width of the GaN (Ge) layers is l_1 (l_2). For $z > z_N$ halfspace filled with GaN.