Journal of Quantitative Spectroscopy & Radiative Transfer 151 (2015) 192–197

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Scattering of on-axis Gaussian beam by an arbitrarily shaped chiral object

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ARTICLE INFO

Article history:
Received 30 April 2014
Received in revised form 28 July 2014
Accepted 2 October 2014
Available online 14 October 2014

Keywords:
Scattering
Gaussian beam
Arbitrarily shaped chiral object

ABSTRACT

Based on a combination of the extended boundary condition method (EBCM) and generalized Lorenz-Mie theory (GLMT), a semi-analytical solution to the scattering of an on-axis Gaussian beam by an arbitrarily shaped chiral object is constructed, by expanding the incident Gaussian beam, scattered fields as well as internal fields in terms of appropriate spherical vector wave functions (SVWFs). The unknown expansion coefficients are determined by using Schelkunoff's equivalence theorem and continuous boundary conditions. Numerical results of the normalized differential scattering cross-section are presented, and the scattering characteristics are discussed concisely.

1. Introduction

The electromagnetic (EM) properties of chiral media have aroused increasing interest over the years, for a variety of applications involving antennas and arrays, antenna radomes, microstrip substrates, and waveguides. One of the basic problems to investigate the interaction between EM waves and chiral media is to describe the EM scattering by chiral objects. For normal incidence of a TE or TM plane wave, analytical solutions have been given by Kluskens et al. to the scattering by a multilayer chiral circular cylinder [1], and by Khatir et al. to the scattering by a chiral elliptic cylinder [2]. Bohren calculated the scattering of light by a sphere with intrinsic optical activity [3]. The method of moments technique has been introduced by Worasawate et al. [4], and the bi-isotropic finite difference time domain technique by Semichaevsky et al. [5], for analyzing the EM plane wave scattering from a homogeneous chiral body. For an incident shaped beam, Yokota et al. studied the scattering of a Hermite-Gaussian beam by a chiral sphere using the relations between the multipole fields and the conventional Hermite-Gaussian beam [6].

First published by Waterman and later studied by so many researchers, the EBCM provides a general formulation for the EM scattering from homogeneous and inhomogeneous arbitrarily shaped obstacles [7–11]. Gouesbet et al. developed the GLMT, with which to deal with the interaction between arbitrary EM shaped beams and some regular bodies [12–16]. In this paper, the use of the EBCM, in combination with the GLMT, enables us to construct a semi-analytical solution to the on-axis Gaussian beam scattering by an arbitrarily shaped chiral object.

This paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the scattered fields of an on-axis Gaussian beam by an arbitrarily shaped chiral object. In Section 3, numerical results of on-axis Gaussian beam scattering properties are presented. Section 4 is the conclusion.

2. Formulation

2.1. Expansions of on-axis Gaussian beam, scattered and internal fields in terms of the SVWFs

As shown in Fig. 1, an arbitrarily shaped chiral object is attached to the Cartesian coordinate system *Oxyz*. An incident Gaussian beam propagates in free space and along

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http://dx.doi.org/10.1016/j.jqsrt.2014.10.001

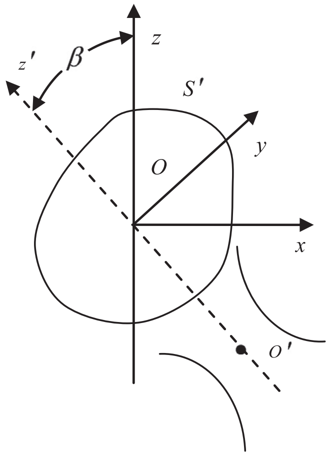

Fig. 1. An arbitrarily shaped chiral object illuminated by an on-axis Gaussian beam.

the axis $O'z'$ in the plane xOz, with the middle of its beam waist located at origin O' on the axis $O'z'$. Origin O has a coordinate z_0 on the axis $O'z'$ (on-axis case), and the angle made by the axis $O'z'$ with the axis Oz is β. In this paper, an $\exp(-i\omega t)$ time variation is assumed.

We have obtained an expansion in [17] of the EM fields of an on-axis incident Gaussian beam (focused TEM_{00} mode laser beam), for the TE mode, in terms of the SVWFs with respect to the system $Oxyz$ [18], in the following form

$$\mathbf{E}^i = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \Big[G_{n,TE}^m \mathbf{M}_{mn}^{r(1)}(k\mathbf{r}) + G_{n,TM}^m \mathbf{N}_{mn}^{r(1)}(k\mathbf{r}) \Big] \tag{1}$$

$$\mathbf{H}^i = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \Big[G_{n,TE}^m \mathbf{N}_{mn}^{r(1)}(k\mathbf{r}) + G_{n,TM}^m \mathbf{M}_{mn}^{r(1)}(k\mathbf{r}) \Big] \tag{2}$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0}$ is the free space wave impedance, and $G_{n,TE}^m$ and $G_{n,TM}^m$ are the expansion or Gaussian beam shape coefficients

$$\begin{bmatrix} G_{n,TE}^m & G_{n,TM}^m \end{bmatrix} = i^n \frac{(n-m)!}{(n+m)!} g_n \begin{bmatrix} \frac{dP_n^m(\cos\beta)}{d\beta} & m\frac{P_n^m(\cos\beta)}{\sin\beta} \end{bmatrix} \tag{3}$$

where g_n, when the Davis-Barton model of the Gaussian beam is used [19], can be described by a simpler expression known as the localized approximation [13,20]

$$g_n = \frac{1}{1+2isz_0/w_0} \exp(ikz_0) \exp\left[\frac{-s^2(n+1/2)^2}{1+2isz_0/w_0} \right] \tag{4}$$

where $s = 1/(kw_0)$, and w_0 is the beam waist radius.

For the TM mode, the corresponding expansions of the Gaussian beam can be obtained only by replacing $G_{n,TE}^m$ in Eqs. (1) and (2) with $iG_{n,TM}^m$, and $G_{n,TM}^m$ with $-iG_{n,TE}^m$.

Following Eqs. (1) and (2), we can correspondingly expand the scattered fields in terms of the SVWFs, as follows:

$$\mathbf{E}^s = E_0 \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \Big[\alpha_{mn} \mathbf{M}_{mn}^{r(3)}(k\mathbf{r}) + \beta_{mn} \mathbf{N}_{mn}^{r(3)}(k\mathbf{r}) \Big] \tag{5}$$

$$\mathbf{H}^s = -iE_0 \frac{1}{\eta_0} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{\infty} (-1)^m \frac{2n+1}{n(n+1)} \Big[\alpha_{mn} \mathbf{N}_{mn}^{r(3)}(k\mathbf{r}) + \beta_{mn} \mathbf{M}_{mn}^{r(3)}(k\mathbf{r}) \Big] \tag{6}$$

where α_{mn} and β_{mn} are the unknown expansion coefficients to be determined.

The constitutive relations for a chiral medium can be written as [21]

$$\mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} + i\kappa \sqrt{\mu_0 \varepsilon_0} \mathbf{H} \tag{7}$$

$$\mathbf{B} = \mu_0 \mu_r \mathbf{H} - i\kappa \sqrt{\mu_0 \varepsilon_0} \mathbf{E} \tag{8}$$

where κ is the chirality parameter.

As discussed in [22], the SVWFs can be combined to represent two eigenwaves, the right and left circularly polarized waves (RCP and LCP waves), in a chiral medium. Then, the EM fields within an arbitrarily shaped chiral object (internal fields) can be expanded in the form

$$\mathbf{E}^w = E_0 \sum_{p=-\infty}^{\infty} \sum_{q=|p|}^{\infty} \Big\{ c_{pq} [\mathbf{M}_{pq}^{r(1)}(k_+\mathbf{r}) + \mathbf{N}_{pq}^{r(1)}(k_+\mathbf{r})] + d_{pq} [\mathbf{M}_{pq}^{r(1)}(k_-\mathbf{r}) - \mathbf{N}_{pq}^{r(1)}(k_-\mathbf{r})] \Big\} \tag{9}$$

$$\mathbf{H}^w = -i\frac{1}{\eta} E_0 \sum_{p=-\infty}^{\infty} \sum_{q=|p|}^{\infty} \Big\{ c_{pq} [\mathbf{M}_{pq}^{r(1)}(k_+\mathbf{r}) + \mathbf{N}_{pq}^{r(1)}(k_+\mathbf{r})] - d_{pq} [\mathbf{M}_{pq}^{r(1)}(k_-\mathbf{r}) - \mathbf{N}_{pq}^{r(1)}(k_-\mathbf{r})] \Big\} \tag{10}$$

where $\eta = \eta_0 \sqrt{\mu_r/\varepsilon_r}$, $k_\pm = k_0(\sqrt{\mu_r \varepsilon_r} \pm \kappa)$.

2.2. Determination of scattered fields based on the EBCM scheme

As discussed in [10], by using Schelkunoff's equivalence theorem and continuous boundary conditions of EM fields the surface currents in terms of the internal fields can be set up to produce null fields inside the chiral object's surface S' and actual scattered fields outside S', and the explicit expressions are described by the following integral equations:

$$\mathbf{E}^i + \oint\!\!\oint_{S'} \Big\{ i\omega\mu \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') \cdot [\hat{n} \times \mathbf{H}^w(\mathbf{r}')] + \nabla \times \overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') \cdot [\hat{n} \times \mathbf{E}^w(\mathbf{r}')] \Big\} dS' = 0 \tag{11}$$

$$\mathbf{E}^s = \oint\!\!\oint_{S'} \Big\{ i\omega\mu \overline{\overline{G}}_0'(\mathbf{r},\mathbf{r}') \cdot [\hat{n} \times \mathbf{H}^w(\mathbf{r}')] + \nabla \times \overline{\overline{G}}_0'(\mathbf{r},\mathbf{r}') \cdot [\hat{n} \times \mathbf{E}^w(\mathbf{r}')] \Big\} dS' \tag{12}$$

where $\hat{n}$ denotes the outward normal to S', $\mathbf{r}' \in S'$, and $\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}')$, $\overline{\overline{G}}_0'(\mathbf{r},\mathbf{r}')$ are the free space dyadic Green's functions given by Tai [23]

$$\overline{\overline{G}}_0(\mathbf{r},\mathbf{r}') = \frac{ik}{4\pi} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{n} (-1)^m \frac{2n+1}{n(n+1)} \times [\mathbf{M}_{mn}^{r(1)}(k\mathbf{r}) \mathbf{M}_{(-m)n}^{r(3)}(k\mathbf{r}') + \mathbf{N}_{mn}^{r(1)}(k\mathbf{r}) \mathbf{N}_{(-m)n}^{r(3)}(k\mathbf{r}')] \tag{13}$$

$$\overline{\overline{G}}_0'(\mathbf{r},\mathbf{r}') = \frac{ik}{4\pi} \sum_{m=-\infty}^{\infty} \sum_{n=|m|}^{n} (-1)^m \frac{2n+1}{n(n+1)} \times [\mathbf{M}_{mn}^{r(3)}(k\mathbf{r}) \mathbf{M}_{(-m)n}^{r(1)}(k\mathbf{r}') + \mathbf{N}_{mn}^{r(3)}(k\mathbf{r}) \mathbf{N}_{(-m)n}^{r(1)}(k\mathbf{r}')] \tag{14}$$

Substituting Eq. (13) into Eq. (11) and then equating the expansion coefficients of $\mathbf{E}^i$ in Eq. (11) and in Eq. (1), we