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Shape optimization of radiant enclosures with specular-diffuse surfaces by means of a random search and gradient minimization

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ABSTRACT

A technique of the shape optimization of radiant enclosures with specular-diffuse surfaces is proposed. The shape optimization problem is formulated as an operator equation of the first kind with respect to a surface to be optimized. The operator equation is reduced to a minimization problem for a least-squares objective shape functional. The minimization problem is solved by a combination of the pure random (or blind) search (the simplest stochastic minimization method) and the conjugate gradient method. The random search is used to find a starting point for the gradient method. The latter needs the gradient of the objective functional. The shape gradient of the objective functional is derived by means of the shape sensitivity analysis and the adjoint problem method. Eventually, the shape gradient is obtained as a result of solving the direct and adjoint problems. If a surface to be optimized is given by a finite number of parameters, then the objective functional becomes a function in a finite-dimensional space and the shape gradient becomes an ordinary gradient. Numerical examples of the shape optimization of "two-dimensional" radiant enclosures with polyhedral specular or specular-diffuse surfaces are given.

1. Introduction

Thermal radiation plays decisive role in many high-temperature setups and systems, such as infrared thermal chambers, furnaces and ovens for materials processing and manufacturing, radiant heaters and reflectors, and solar collectors. Enclosure geometry significantly influences radiative heat transfer. Therefore, the shape optimization of radiant enclosures is of fundamental importance.

Geometric (shape) optimization of radiant enclosures was considered in Refs. [1–5], where two-dimensional problems were considered and design problems were reduced to minimization of least-squares objective functions. In Ref. [1] optimal boundary-value and shape design problems were considered. Design parameters were positions of three tube radiators in radiant heaters and uniform radiative heat fluxes on the surfaces of the radiators. Design objective was a uniform radiative heat flux on a design surface. The radiant heaters were considered both with a reflector and without it. The minimization problems were solved by a modification of the Nelder–Mead simplex method. In Refs. [2,3] geometric optimization of radiant enclosures with transparent medium was considered. In Ref. [2] surfaces of the enclosure were diffuse, while in Ref. [3] the enclosures contained plane specular surfaces. Surfaces to be optimized were described parametrically by few parameters. Objective functions were

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Nomenclature

A	operator of the direct problem (Eq. (3.1), Fig. 2)
A'	derivative of A with respect to variations of S_o (see Eq. (4.9)), the operator $A'(S_o)$ is determined by the sensitivity problem
D	domain in $\mathbb{R}^3$
D_t	perturbed domain ($D_t=\boldsymbol{\varphi}_t(D)$)
$\mathrm{d}\mathbf{r}$	volume integration
$\mathrm{d}s$	surface integration
$\mathrm{d}\boldsymbol{\Omega}$	integration on the unit sphere, i.e., $\mathrm{d}\boldsymbol{\Omega}=\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$
I	radiation intensity ($I\equiv I(\mathbf{r},\boldsymbol{\Omega})$)
$I_{\mathrm{b,s}}$	black-body intensity on a surface ($I_{\mathrm{b,s}}\equiv I_{\mathrm{b,s}}(\mathbf{r})$)
I_t	solution to the direct problem in D_t ($I_t\equiv I_t(\tilde{\mathbf{r}},\boldsymbol{\Omega})$, $\tilde{\mathbf{r}}\in D_t$)
I^t	$I^t\equiv I^t(\mathbf{r},\boldsymbol{\Omega})=I_t(\boldsymbol{\varphi}_t(\mathbf{r}),\boldsymbol{\Omega})$, $\mathbf{r}\in D$ (see the caption of Fig. 3)
I'	shape derivative of I (Eq. (4.8))
$\dot{I}$	material derivative of I (Eq. (4.12))
I^*	"adjoint intensity", solution to the adjoint problem (C.11) and (C.12) ($I^*\equiv I^*(\mathbf{r},\boldsymbol{\Omega})$)
J	objective functional (Eq. (3.4))
J'	shape gradient of J (Eq. (4.11))
$\hat{J}$	"finite-parametric" objective functional (Eq. (5.2))
$\mathbf{J}_{\ldots}$	Jacobian matrix, e.g., $\mathbf{J}_{\boldsymbol{\varphi}_t}$, $\mathbf{J}_{\boldsymbol{\psi}_t}$, $\mathbf{J}_{\mathbf{v}}$
$(\mathbf{J}_{\ldots})_\tau$	tangential Jacobian matrix, e.g., $(\mathbf{J}_{\mathbf{v}})_\tau$, $(\mathbf{J}_{\mathbf{n}})_\tau$, $(\mathbf{J}_{\mathbf{g}})_\tau$ (Eq. (D.5))
$\mathbf{n}$	outward unit normal to a surface ($\mathbf{n}\equiv\mathbf{n}(\mathbf{r})$)
$\tilde{\mathbf{n}}$, $\mathbf{n}_t$	outward unit normal to the surface S_t ($\tilde{\mathbf{n}}\equiv\mathbf{n}(\tilde{\mathbf{r}})$, $\mathbf{n}_t\equiv\mathbf{n}_t(\mathbf{r})=\mathbf{n}(\boldsymbol{\varphi}_t(\mathbf{r}))$, Fig. 19)
$\mathbf{n}'$	$\mathbf{n}'\equiv\mathbf{n}'(\mathbf{r})\overset{\mathrm{def}}{=}(\mathrm{d}/\mathrm{d}t)\mathbf{n}_t\vert_{t=0+}$ (Eq. (A.8), Fig. 19), note that $\mathbf{n}'\cdot\mathbf{n}=0$
$\mathbf{p}$	parameters, determining a surface to be optimized ($\mathbf{p}=(p_1,\ldots,p_m)$)
q	radiative heat flux ($q\equiv q(\mathbf{r})$, $\mathbf{r}\in S$)
$q_{\mathrm{b,s}}$	black-body flux on a surface ($q_{\mathrm{b,s}}\equiv q_{\mathrm{b,s}}(\mathbf{r})=\pi I_{\mathrm{b,s}}(\mathbf{r})$)
$\overline{q}_{\mathrm{d}}^{\mathrm{inc}}$	radiative heat flux, specified on the design surface ($\overline{q}_{\mathrm{d}}^{\mathrm{inc}}\equiv\overline{q}_{\mathrm{d}}^{\mathrm{inc}}(\mathbf{r})$)
$q^{t,\mathrm{inc}}$	incident flux, corresponding to I^t (Eq. (A.7))
$\dot{q}^{\mathrm{inc}}$	incident flux, corresponding to $\dot{I}$ (Eq. (A.12))
$q^{\prime,\mathrm{inc}}$	incident flux, corresponding to I' (Eq. (4.7))
$q^{*,\mathrm{inc}}$	incident flux, corresponding to I^* (Eq. (C.13))
$q^{*,\mathrm{out}}$	see Eq. (C.5)
$\mathbf{r}$	point in D
$\tilde{\mathbf{r}}$	point in D_t
S	surface, boundary of D
S_t	perturbed surface, boundary of D_t ($S_t=\boldsymbol{\varphi}_t(S)$)
t	parameter of the transformation $\boldsymbol{\varphi}_t$
$\mathbf{u}_{\mathrm{o}}$	vector field (Eqs. (C.14)–(C.16))
$\mathbf{v}$	velocity field ($\mathbf{v}\equiv\mathbf{v}(\mathbf{r})$, Eq. (4.4))
$\mathbf{v}_{\mathrm{o}}$	$\mathbf{v}_{\mathrm{o}}\equiv\mathbf{v}\vert_{S_{\mathrm{o}}}$

Greek symbols

$\boldsymbol{\alpha}$	$\boldsymbol{\alpha}(\mathbf{r})=\int_{\Omega_{\mathrm{n}}>0}\boldsymbol{\Omega}_\tau I(\mathbf{r},\boldsymbol{\Omega})\,\mathrm{d}\boldsymbol{\Omega}$, $\mathbf{r}\in S$ (Eq. (A.13))
ε	surface emissivity ($\varepsilon\equiv\varepsilon(\mathbf{r})$, $\mathbf{r}\in S$)
η	test function (Eq. (A.3), $\eta\equiv\eta(\mathbf{r},\boldsymbol{\Omega})$)
θ	polar angle
κ	absorption coefficient
ρ	surface reflectivity ($\rho\equiv\rho(\mathbf{r})$, $\mathbf{r}\in S$)
ϕ	azimuthal angle
$\boldsymbol{\varphi}_t$	mapping, defining a perturbation of the domain D (Eq. (4.4))
$\boldsymbol{\chi}$	vector field, $\boldsymbol{\chi}\equiv\boldsymbol{\chi}(\mathbf{r},\boldsymbol{\Omega})$, Eqs. (B.4) or (B.7), (B.8) and (B.11)
$\boldsymbol{\Psi}$	direction ($\boldsymbol{\Psi}\in\mathbb{S}^2$)
$\boldsymbol{\psi}_t$	inverse mapping ($\boldsymbol{\psi}_t=\boldsymbol{\varphi}_t^{-1}$)
$\boldsymbol{\Omega}$	direction ($\boldsymbol{\Omega}\in\mathbb{S}^2$, $\boldsymbol{\Omega}=(\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$)
$\hat{\boldsymbol{\Omega}}$	direction of incident radiation to be reflected into $\boldsymbol{\Omega}$ at the point $\mathbf{r}$ of the surface S ($\hat{\boldsymbol{\Omega}}\equiv\hat{\boldsymbol{\Omega}}(\mathbf{r})$, Eq. (2.3), Fig. 1)
$\hat{\boldsymbol{\Omega}}_t$	direction of incident radiation to be reflected into $\boldsymbol{\Omega}$ at the point $\tilde{\mathbf{r}}$ of the surface S_t ($\hat{\boldsymbol{\Omega}}_t\equiv\hat{\boldsymbol{\Omega}}_t(\tilde{\mathbf{r}})$, Eq. (A.6), Fig. 19)
$\hat{\boldsymbol{\Omega}}'$	$\hat{\boldsymbol{\Omega}}'\equiv\hat{\boldsymbol{\Omega}}'(\mathbf{r})=(\mathrm{d}/\mathrm{d}t)\hat{\boldsymbol{\Omega}}_t\vert_{t=0+}$ (Eq. (A.9), Fig. 19)
Ω_{n}	$\Omega_{\mathrm{n}}=\boldsymbol{\Omega}\cdot\mathbf{n}$, $\Omega_{\tilde{\mathrm{n}}}=\boldsymbol{\Omega}\cdot\tilde{\mathbf{n}}$, $\Omega_{\mathrm{n}_t}=\boldsymbol{\Omega}\cdot\mathbf{n}_t$
$\boldsymbol{\omega}$	$\boldsymbol{\omega}\equiv\boldsymbol{\omega}(\mathbf{r},\boldsymbol{\Omega})$, Eq. (A.11)

Other symbols

div_τ	tangential divergence (Eq. (D.3))
∇	gradient with respect to the space variables, i.e., $\nabla=(\partial/\partial x\ \partial/\partial y\ \partial/\partial z)^T$, if $\mathbf{r}=(x,y,z)$
∇_τ	tangential gradient (Eq. (D.1))
$\nabla_{\boldsymbol{\Omega}}I$	gradient of the intensity with respect to $\boldsymbol{\Omega}$, angular gradient (Eq. (A.10))
$\mathbb{R}^n$	n-dimensional Euclidean space
$\mathbb{S}^2$	unit sphere in $\mathbb{R}^3$

Subscripts

b	black-body
d	design surface (S_{d}) or diffuse (ρ_{d})
f	"optimization- and design-free"
h	heater
n	the normal component of a vector, e.g., $v_{\mathrm{n}}=\mathbf{v}\cdot\mathbf{n}$, $\Omega_{\mathrm{n}}=\boldsymbol{\Omega}\cdot\mathbf{n}$
o	surface to be optimized
τ	the tangential component of a vector, e.g., $\mathbf{v}_\tau=\mathbf{v}-v_{\mathrm{n}}\mathbf{n}$, $\boldsymbol{\Omega}_\tau=\boldsymbol{\Omega}-\Omega_{\mathrm{n}}\mathbf{n}$ (Fig. 19)
s	surface ($I_{\mathrm{b,s}}$, $q_{\mathrm{b,s}}$) or specular (ρ_{s})

Superscripts

inc	incident radiation
out	outgoing radiation (radiosity)
T	matrix transposition
$*$	adjoint

Inner products and norms

$\vert\cdot\vert$	the Euclidean norm (length) ($\vert\mathbf{x}\vert=\sqrt{\mathbf{x}\cdot\mathbf{x}}$)
$\langle\cdot,\cdot\rangle_{S_{\mathrm{d}}}$	$\langle u,v\rangle_{S_{\mathrm{d}}}=\int_{S_{\mathrm{d}}}uv\,\mathrm{d}s$
$\Vert\cdot\Vert_{S_{\mathrm{d}}}$	$\Vert u\Vert_{S_{\mathrm{d}}}^2=\int_{S_{\mathrm{d}}}\vert u\vert^2\,\mathrm{d}s$
$\langle\cdot,\cdot\rangle_{S_{\mathrm{o}}}$	$\langle\mathbf{u},\mathbf{v}\rangle_{S_{\mathrm{o}}}=\int_{S_{\mathrm{o}}}\mathbf{u}\cdot\mathbf{v}\,\mathrm{d}s$