Journal of Quantitative Spectroscopy & Radiative Transfer 147 (2014) 252–260

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Scattering of evanescent wave by multiple parallel infinite cylinders near a surface

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ARTICLE INFO

Article history:
Received 28 March 2014
Received in revised form
10 June 2014
Accepted 11 June 2014
Available online 19 June 2014

Keywords:
Evanescent wave
Surface wave interaction
Optical tunneling
Infinite cylinder
Far-field scattering

ABSTRACT

This paper presents an exact analytical solution for the scattering of evanescent wave by an arbitrary collection of parallel infinite cylinders located near the surface of an optically denser substrate. The evanescent wave is generated by total internal reflection of the source wave propagating within the substrate at greater than the critical angle. The source wave is arbitrarily polarized and propagates in the plane perpendicular to the axes of the cylinders. The theoretical formulation utilizes Hertz potentials, which accounts for the near-field scattering between the cylinders and Fresnel reflection of angular distribution of scattered waves from the surface. Analytical formulas are derived for the electric and magnetic fields and Poynting vector of the scattered radiation in the near field, as well as their asymptotic forms in the far-field. Numerical results are presented to illustrate the scattering of evanescent wave by several configurations of cylinders located near the surface.

1. Introduction

There are significant interests in the scattering of evanescent wave by particles at or near a surface due to its relevance to near-field microscopy. These applications include scanning optical microscopy, scanning electron microscopy, scanning tunneling microscopy, etc. The particles are either located on, or suspended in an aqueous solution above, the surface of the substrate that has a higher refractive index. A light source illuminates the interface from within the substrate at an angle greater than critical angle. Total internal reflection occurs as a result, which gives rise to a transmitted wave that decays exponentially with distance from the surface. This decaying wave is called an evanescent wave. Optical tunneling occurs when a particle is placed near the surface. The size, composition, and position of the particle can be determined by measuring the intensity and polarization of the scattered wave.

Optical microscopy applications using evanescent wave have been designated various names in the literature. Called total internal reflection microscopy (TIRM), Prieve et al. [1,2] utilized the technique to study the equilibrium and dynamic behavior of charged particles near flat surfaces. The vertical motion of the particle above the interface is determined by measuring the intensity of the scattered light. In studies on the near-wall dynamics of colloidal systems, the penetration depth of the evanescent wave is tuned by changing the angle of incidence of the source beam. This technique is called Evanescent wave dynamic light scattering (EWDLS), which enables probing of the colloidal system at different length scales [3,4]. Aslan et al. [5] utilized the polarization of the scattered wave to characterize the particles and called this approach

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http://dx.doi.org/10.1016/j.jqsrt.2014.06.006
0022-4073/

elliptically polarized surface-wave scattering (EPSWS). Analyses of evanescent scattering generally utilize numerical and semi-analytical methods. Numerical analyses based on the discrete source and T-matrix methods have been used to treat the case of spherical particle [6–8]. Many literature citations can be found in the references of the above papers, which are not repeated here.

Characterization of cylindrical obstacles on a surface is an important fundamental problem that has many applications. The objective of this paper is to present an analytical solution based on Hertz potentials for evanescent scattering by multiple infinite parallel cylinders located near the surface of a substrate. The present problem shares some commonalities with that of scattering by cylinders buried in a half space [9–11], because both cases involve scattering near a boundary. The commonalities allow the adaption of the theoretical treatment on near-field scattering between cylinders and Fresnel reflection of scattered waves from the half space interface. However, the problem of cylinders buried in a half space differs from the present study in several major aspects. The differences include: (1) the source wave was transmitted from a lower refractive index medium into a medium of higher refractive index; (2) the cylinders were located in the higher refractive index medium; and (3) backscattering transmitted out of the half space containing the cylinders was of interest. The conditions of the present problem are exactly opposite to (1)–(3), and the quantity of interest is the forward scattering in the same half space where the cylinders are located.

In the present problem the source wave is arbitrarily polarized and propagates within the substrate in the plane perpendicular to the axes of the cylinders. The source wave propagates at greater than the critical angle, thus giving rise to an evanescent wave across the interface and incident on the cylinders. For generality each cylinder is radially stratified and distinct, and no restriction is placed on their size and location. In the following sections the theoretical formulation is first presented, followed by numerical examples to illustrate scattering of evanescent wave by different configurations of cylinders.

2. Theory

Fig. 1 shows a schematic diagram of the present problem. An arbitrary configuration of infinite cylinders, each of which is distinct and radially stratified, is aligned parallel to the Z-axis near the medium 1–2 interface. Media 1 and 2 are both non-absorbing dielectrics with real refractive indexes $\tilde{m}_1$ and $\tilde{m}_2$, respectively, and $\tilde{m}_1 > \tilde{m}_2$. In typical optical microscopy systems medium 1 can be a glass substrate and medium 2 is either air or an aqueous solution. The source wave propagates in medium 1 in the XY plane at an angle θ_1 inclined from the $-X$ axis.

The pertinent scattering phenomena involving the cylinders are depicted in Fig. 2. The primary incident wave on the cylinders arises from the transmitted source wave. The secondary and tertiary incident waves include the near-field scattered waves from other cylinders and reflected waves from the substrate's surface due to scattered waves from all cylinders. The total electric (E) and magnetic (H) fields in the vicinity of a cylinder can be written as

$$\begin{pmatrix} \vec{E}_{\psi j} \\ \vec{H}_{\psi j} \end{pmatrix} = \begin{pmatrix} \vec{E}^{+}_{\psi j} \\ \vec{H}^{+}_{\psi j} \end{pmatrix} + \sum_{k \neq j}^{N} \begin{pmatrix} \vec{E}^{s}_{\psi ,kj} \\ \vec{H}^{s}_{\psi ,kj} \end{pmatrix} + \sum_{k=1}^{N} \begin{pmatrix} \vec{E}^{r+}_{\psi ,kj} \\ \vec{H}^{r+}_{\psi ,kj} \end{pmatrix} + \begin{pmatrix} \vec{E}^{s}_{\psi j} \\ \vec{H}^{s}_{\psi j} \end{pmatrix} \quad (1)$$

where the first 3 terms on the right hand side correspond to the respective incident waves, and the last term refers to the scattered wave from the cylinder. The subscript $\psi(=u,v)$ refers to the polarization of the source wave, which can be magnetic (u) or electric (v) mode. These fields satisfy Maxwell's relations given in terms of the Hertz potentials (u,v) as [12]

$$\vec{E} = \nabla \times (\vec{e}_Z v) + \frac{i}{k}\nabla \times \nabla \times (\vec{e}_Z u) \quad (2)$$

$$\vec{H} = -\frac{\tilde{m}}{\mu}\nabla \times (\vec{e}_Z u) + \frac{i}{\mu k_o}\nabla \times \nabla \times (\vec{e}_Z v) \quad (3)$$

where $\tilde{m}$ is refractive index, μ is the permeability, $k\,(=\tilde{m}k_o)$ is the propagation constant of the medium, and k_o is that for free space. The formulation for each of the terms in Eq. (1) is described in the following sections.

2.1. Source wave in substrate

The primary incident wave on the cylinders arises from the source wave transmitted across the media 1–2 interface. The Hertz potential $\psi\ (=u,v)$ of the source wave in medium 1 can be written in terms of the complex amplitudes α^i_ψ as

$$\psi^i = \alpha^i_\psi \exp(-i\vec{k}^i \cdot \vec{R}_P) \quad (4)$$

where $\vec{R}_P$ is the radial vector to point P, and

$$\vec{k}^i = k_1(\cos\theta_1 \vec{e}_X - \sin\theta_1 \vec{e}_Y) \quad (5)$$

is the propagation vector in the substrate. The source wave for each polarization mode in medium 1 is formulated by

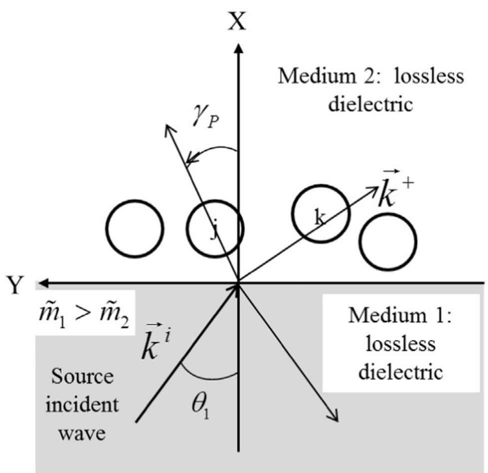

Fig. 1. Schematic diagram of the present problem.