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Optical tweezers: Theory and modelling

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ABSTRACT

Since their development in the 1980s, optical tweezers have become a widely used and versatile tool in many fields. Outstanding applications include the quantitative measurement of forces in cell biology and biophysics. Computational modelling of optical tweezers is a valuable tool in support of experimental work, especially quantitative applications. We discuss the theory, and the theoretical and computational modelling of optical tweezers.

1. Introduction

Before the 1970s, there seemed to be little, if any, prospect of terrestrial application of optical forces. That optical forces could be important in astronomical situations was known [79,143], because free from frictional forces, small accelerations could result in large changes in velocity over time, or in stellar atmospheres where irradiances could be extremely high. Indeed, detailed proposals for solar sail driven spacecraft had been made as early as 1924 [160]. However, the minuteness of optical forces appeared to condemn them to remain a subject for "heroic experiments" (such as the first experimental measurements by [95,123,124]). However, from the early 1970s onwards, optical forces proved capable of manipulating small objects, and two branches of technology developed: atom cooling and trapping, which led to Nobel Prize in Physics in 1997 [27,28,139] and 2001 [31,89], and optical tweezers, or the single-beam gradient trap, where a single tightly focussed laser beam is used to three-dimensionally trap microscopic particles. Optical tweezers were immediately attractive in biological research, due to the ability to trap and move microorganisms without physical contact [13,176], which can even allow manipulation of organelles in live cells [76]. Further, the ability to measure small forces—from femtonewtons to some tens of piconewtons—has made optical tweezers a star player in quantitative biophysics and mechanobiology [120]. The original modern papers on optical forces [6] and optical tweezers [9] and historical perspectives by the pioneer of the field [7,8] provide a basis for a historical overview of the topic. In addition, there is a useful bibliography covering the first few decades of optical tweezers [92]. Since we are discussing the theory and modelling of optical tweezers, we can point out some interesting and useful early theoretical work. Roosen and Imbert [146], Roosen et al. [145], and Roosen [144] carried out early work (following Ashkin's original experiments, but predating optical tweezers). Application of full-wave theories to modelling optical trapping closely followed the invention of optical tweezers; some key theoretical papers from the first decade of optical tweezers are by Barton et al. [11,12], Gussgard et al. [72], Visscher

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and Brakenhoff [165,166], Wright et al. [173], and Ren et al. [142]. An interesting combination of theory and experiment is the early work using optical levitation to test scattering theory by Grehan and Gouesbet [68], Guilloteau et al. [70].

The chief difficulty in practical application of optical forces is that the ratio of momentum flux to energy flux is very small. These are related by the speed of light, such that the momentum flux p of a collimated beam, or ray of light, or plane wave, of power P is $p = nP/c$, where n is the refractive index, and c is the speed of light in free space. This means that the maximum force obtainable in air is less than 10 nN/W. Indeed, the lack of successful laboratory measurements until the beginning of the twentieth century made many skeptical of even the existence of such forces [44], despite earlier theoretical work suggesting their existence [41,14]. Early experimental efforts failed—John Michell demonstrated only the destruction of his experimental apparatus by concentrated sunlight [75] (this work was, however, very fruitful as it led to the development of the torsion balance), and the Crookes radiometer [33–35] demonstrated thermal, rather than optical, forces. Part of the difficulty was the lack of a complete theory of optics, until Maxwell's development of electromagnetic theory [110,111], with further clarification of the transport of momentum by electromagnetic fields and light following shortly [140,10,77]. Interestingly, the same result was obtained on thermodynamic grounds by Umov [161].

Even after the theoretical basis was known, and the first measurements had been made, practical applications appeared to be far-off, perhaps even infinitely far. Terrestrial application requires other forces such as gravity, friction, viscous drag, and Brownian motion to be overcome. For a 1 W beam to be able to lift a solid particle, it would need to be smaller than 100 μm in radius, for typical densities. This, in turn, requires the 1 W beam to be focussed to approximately the size of the particle, or smaller. In the absence of a coherent light source, the irradiance at the focal spot is limited by the *brightness theorem*—the irradiance at the focus cannot exceed the irradiance at the source, which follows from Liouville [97]. To achieve the required irradiance above, i.e., 1 W focussed into an area of 10^{-8} m^2, would require a blackbody with a temperature of over 6000 K.

However, the development of the laser allowed these limits to the irradiance achievable by focussing light from extended sources to be overcome. Ashkin [7,8] realised that the laser enabled greater irradiances to be achieved, which should be sufficient to levitate small particles against gravity [6]. These traps were based on radiation pressure forces, where the beam pushes the particle in the direction of propagation. During these experiments, it was noticed that particles were trapped transversely within the beams—they were attracted to the regions of maximum electric field amplitude. These forces were manifestations of the *gradient force*, and Ashkin realised that it would be possible to produce a gradient force large enough to overcome the radiation pressure force (or *scattering force*), allowing three-dimensional trapping by a single laser beam [9]. The critical ingredient was tightly focussing the beam so that the field gradient is sufficiently large in all directions from the focus. This requires a high-numerical aperture lens; a microscope objective is ideal, and a typical optical tweezers apparatus essentially consists of a laser beam focussed by a microscope, as shown in Fig. 1. The microscope objective conveniently allows the user to observe the trapped particle.

Once one accepts that light carries momentum, the existence of radiation pressure forces which push particles in the direction of propagation due to reflection or absorption follows naturally. The gradient force, which can act against the propagation of the beam, is more mysterious when first encountered. There are two simple qualitative explanations. The first is that the electric field of the beam induces a dipole moment in the trapped particle, which is then attracted to the region of highest field, in the same way as a steel ball bearing is attracted to a magnet. The second explanation is based on the momentum of a converging or diverging beam. The momentum of a converging or diverging beam is less than that of a collimated beam, since, for any part of the beam, only the component of the momentum parallel to the beam axis contributes to the total momentum. The more converging or diverging the beam, the lower the momentum, while the more collimated, the higher the momentum. Thus, if a trapped particle makes the beam more convergent or divergent, it reduces the momentum of the beam, and is pushed in the direction of propagation. If, on the other hand, it makes the beam more collimated, the force opposes the direction of propagation. A typical particle can be considered as a weak positive lens, and if the beam is converging (i.e., if the

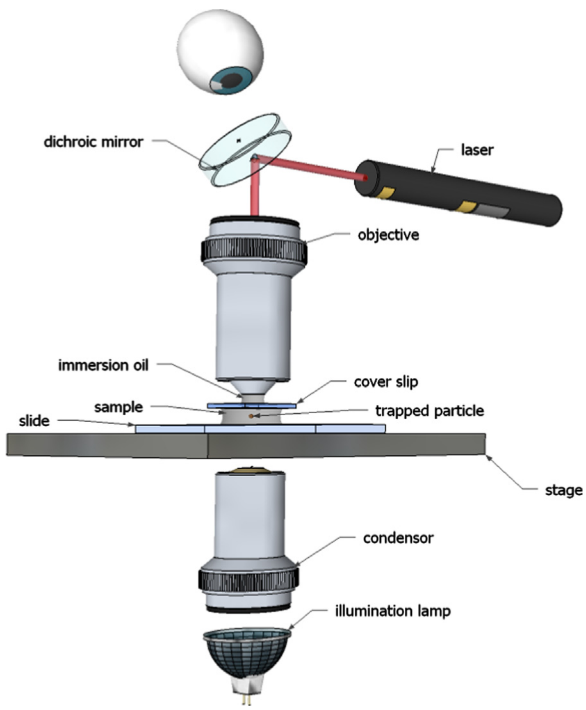

Fig. 1. A typical optical tweezers setup. A dichroic mirror in the microscope reflects incoming laser light towards the sample; and allows illumination light from the condenser to pass through to the eyepiece or camera. An oil layer is used between the lens and the cover slip, in order to allow for the use of numerical apertures greater than one.