Journal of Quantitative Spectroscopy & Radiative Transfer 146 (2014) 290–294

Contents lists available at ScienceDirect

Journal of Quantitative Spectroscopy & Radiative Transfer

journal homepage: www.elsevier.com/locate/jqsrt

Fractal signatures in analogs of interplanetary dust particles

Nisha Katyal [a,*], Varsha Banerjee [b], Sanjay Puri [a]

[a] *School of Physical Sciences, Jawaharlal Nehru University, New Delhi 110067, India*
[b] *Department of Physics, Indian Institute of Technology, Hauz Khas, New Delhi 110016, India*

ARTICLE INFO

Article history:
Received 15 October 2013
Received in revised form
23 January 2014
Accepted 25 January 2014
Available online 31 January 2014

Keywords:
Silicate cores
Interplanetary dust particles
Structure factor
Mass fractals
Diffusion limited aggregation

ABSTRACT

Interplanetary dust particles (IDPs) are an important constituent of the earth's stratosphere, interstellar and interplanetary medium, cometary comae and tails, etc. Their physical and optical characteristics are significantly influenced by the morphology of silicate aggregates which form the core in IDPs. In this paper we reinterpret scattering data from laboratory analogs of cosmic silicate aggregates created by Volten et al. (2007) [1] to extract their morphological features. By evaluating the structure factor, we find that the aggregates are mass fractals with a mass fractal dimension $d_m \simeq 1.75$. The same fractal dimension also characterizes clusters obtained from *diffusion limited aggregation* (DLA). This suggests that the analogs are formed by an irreversible aggregation of stochastically transported silicate particles.

1. Introduction

Fractal geometries provide a description for many forms in nature such as coastlines, trees, blood vessels, fluid flow in porous media, burning wavefronts, dielectric breakdown, diffusion-limited-aggregation (DLA) clusters, bacterial colonies, and colloidal aggregates. [2–4]. They exhibit self-similar and scale-invariant properties at all levels of magnification and are characterized by a non-integer fractal dimension. These features arise because the underlying processes have an element of stochasticity in them. Such processes play an important role in shaping the final morphology, and their origin is distinctive in each physical setting.

Irregular and rough aggregates have also been observed in the astronomical context. Naturally found cosmic dust aggregates, known as interplanetary dust particles (IDPs), are collected in earth's lower stratosphere. They are formed when dust grains collide in a turbulent circumstellar dust cloud, such as the solar nebula, and are an important constituent of the interstellar medium, interplanetary medium, cometary comae and tails, etc. Mass spectroscopy analysis of IDPs has revealed that their primary constituents are (i) silicates of Fe, Mg, Al and Ca, (ii) complex organic molecules of C, H, O and N, (iii) small carbonaceous particles of graphite, coal and amorphous carbon and (iv) ices of CO_2, H_2O and NH_3 [5–9]. Among these, there is an exclusive abundance of silicates which aggregate to form particle *cores*. They have been described as *fluffy, loosely structured particles with high porosity*. The other constituents contribute to the outer covering or the *mantle* and are usually contiguous due to flash heating from solar flares and atmospheric entry [10]. The core, being deep inside retains its morphology. The latter is believed to have a fractal organization characterized by a fractal dimension, but this belief is not on firm grounds as yet [11,12]. As the core morphology affects the physical and optical characteristics of IDPs, its understanding has been the focus of several recent works [13–18].

Two classes of stochastic fractals are found in nature. The first class is that of *surface fractals* whose mass M scales with the radius of gyration R in a Euclidean fashion, i.e., $M \sim R^d$, where d is the dimensionality. However, the surface area S increases with the radius as $S \sim R^{d_s}$, where d_s is the surface fractal dimension and $d-1 \le d_s < d$ [19]. Interfaces generated in fluid flows, burning wavefronts, dielectric breakdown

* Corresponding author.
E-mail address: nishahansraj@gmail.com (N. Katyal).

0022-4073/$ - see front matter © 2014 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.jqsrt.2014.01.017

and deposition processes are examples of surface fractals. The second class is that of *mass fractals* which obey the scaling relationship, $M \sim R^{d_m}$, where d_m is the mass fractal dimension and $1 \le d_m < d$. Examples of mass fractals are DLA clusters, bacterial colonies and colloidal aggregates. Further, in many situations, mass fractals are bounded by surface fractals [2–4]. As a matter of fact, the above mass fractals belong to this class.

There are many unanswered questions in the context of fluffy cores or silicate aggregates of IDPs. For example, are they mass fractals, bounded by surface fractals? What is their mass and surface fractal dimension? What kind of aggregation mechanisms are responsible for this morphology? What are the consequences of fractal organization on the evolution of clusters? In this paper, we provide answers to some of these questions using the *real-space correlation function* $C(r)$ and the *momentum-space structure factor* $S(k)$. Smooth morphologies are characterized by the *Porod law* [20,21]. The *signature* of fractal domains and interfaces is a *power-law decay* with non-integer exponents in $C(r)$ and $S(k)$. As typical experimental morphologies are smooth on some length scales and fractal on others, the behaviors of $C(r)$ vs. r and $S(k)$ vs. k are characterized by *cross-overs* from one form to another. We identify these features in laboratory analogs of cores of IDPs created by Volten et al. using magnesio-silica grains, by reinterpreting their light-scattering data [1]. We find that these aggregates are mass fractals with a fractal dimension $d_m \simeq 1.75$. The same fractal dimension characterizes diffusion limited aggregation (DLA). We therefore infer that aggregation mechanisms of silicate cores in IDPs are stochastic and irreversible as in DLA.

This paper is organized as follows. In Section 2, we describe the tools for morphology characterization and their usage to obtain mass and surface fractal dimensions. In Section 3, we describe the experimental analogs of silicate cores in IDPs and obtain the structure factor from their light scattering data to extract fractal properties. In Section 4, we present a simulation of the DLA cluster, and the evaluation of its structure factor and the corresponding mass fractal dimension. Finally, we conclude with a summary and discussion of our results in Section 5.

2. Tools for morphology characterization

A standard tool to obtain information about sizes and textures of domains and interfaces is the two-point spatial correlation function [21]

$$C(r) = \langle \psi(\vec{r_i})\psi(\vec{r_j})\rangle - \langle \psi(\vec{r_i})\rangle\langle \psi(\vec{r_j})\rangle, \tag{1}$$

where $\psi(\vec{r_i})$ is an appropriate order parameter and $r = |\vec{r_i} - \vec{r_j}|$. (We assume the system to be translationally invariant and isotropic.) The angular brackets denote an ensemble average.

The scattering of a plane wave by a rough morphology can yield useful information about the texture of the domains and interfaces in it. Thus, small-angle scattering experiments (using X-rays, neutrons, etc.) can be used to probe their nature. The intensity of the scattered wave in these experiments yields the momentum-space structure factor, which is the Fourier transform of the correlation function [20–23]

$$S(\vec{k}) = \int d\vec{r}\; e^{i\vec{k}\cdot\vec{r}}\, C(\vec{r}), \tag{2}$$

where $\vec{k}$ is the wave-vector of the scattered beam. The properties of $C(r)$ and $S(k)$ provide deep insights into the nature of the scattering morphology.

Consider a domain of size ξ formed by spherical particles of size a, as depicted schematically in Fig. 1(a). The typical interfacial width w is also indicated. This prototypical morphology could represent a colloidal aggregate, soot particles, a DLA cluster, etc. The correlation function for such a morphology can be approximated by

$$1 - C(r) = \overline{C}(r) \simeq \begin{cases} Ar^{\alpha}, & w \ll r \ll \xi, \\ Br^{\beta}, & r \ll w \ll a, \\ Cr^{\gamma}, & r \ll a. \end{cases} \tag{3}$$

The first term conveys information about the domain texture probed by length scales $w \ll r \ll \xi$. If the domain has no internal structure, $\alpha = 1$ signifying the Porod decay [20,21]. For a fractal domain, on the other hand, $\alpha = d_m - d$, where d_m is the mass fractal dimension [22,23]. The second term conveys information about the properties of the interface, probed by lengths $a \ll r \ll w$. For fractal interfaces, $0 \le \beta < 1$, and β is related to the fractal dimension as $d_s = d - \beta$ [24]. The third term is significant only if the building blocks are particles of diameter a. In that case, $\gamma = 1$ for $r \lesssim a$, yielding the Porod regime at a microscopic length scale.

In the Fourier space, Eq. (3) translates into the following power-law behavior of the structure factor:

$$S(k) \simeq \begin{cases} \tilde{A}k^{-(d+\alpha)}, & \xi^{-1} \ll k \ll w^{-1}, \\ \tilde{B}k^{-(d+\beta)}, & w^{-1} \ll k \ll a^{-1}, \\ \tilde{C}r^{-(d+\gamma)}, & a^{-1} \ll k. \end{cases} \tag{4}$$

The Porod decay of the form $k^{-(d+1)}$ in the scattered intensity is typical of smooth domains or sharp interfaces [20,21]. A deviation from this behavior to $S(k) \sim k^{-(d \pm \theta)}$ is indicative of a fractal structure in the domains or interfaces. When physical structures have multiple length-scales, one or more terms in Eqs. (3) and (4) may contribute. Their presence is characterized by cusps in the correlation function, and corresponding power-laws in the structure factor [25].

We illustrate the power laws and cross-overs discussed above in the context of the 2-d morphology depicted in Fig. 1(a). It should be noted that both the domain and the interfacial boundary in this schematic are rough, self-similar fractals. The structure factor $S(k)$ vs. k for this morphology obtained from the Fourier transform of the spherically averaged correlation function $C(r)$ vs. r is plotted in Fig. 1(b) on a log–log scale. This function exhibits two distinct regimes over large and small values of k as seen from the best fit lines: a *power-law decay* with $S(k) \sim k^{-1.71}$ for $\xi^{-1} \ll k \ll w^{-1}$ and the *Porod decay* with $S(k) \sim k^{-3}$ for $a^{-1} \ll k$. With reference to Eqs. (3) and (4), the power law decay signifies a fractal domain morphology with a mass fractal dimension $d_m \approx 1.71$ while the Porod decay is due to the smooth surface of the particles. The structure factor corresponding to wave vectors in the interval $w^{-1} \ll k \ll a^{-1}$ is due to scattering from the rough fractal