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Scattering of an oblique incident focused Gaussian beam by a spheroidal particle

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ABSTRACT

Based on the expansion of a Gaussian beam in spheroidal coordinates in general case of oblique illumination, a theoretical procedure to determine the scattered fields of spheroid obliquely illuminated by a Gaussian beam is presented. Specific attention is paid to the study of scattering properties of a spheroidal particle from an obliquely incident Gaussian beam. The calculated results for spheroid are compared with those from the surface integral equation method, and very good agreements are observed. Numerical results concerning the influences of shaped beam parameters (beam waist radius, incident angle) as well as spheroid parameters (major axis, minor axis, refractive index, size parameter) on the scattering properties are presented. These results can be used as a reference for other numerical methods to analyze the light scattering by non-spherical particles illuminated by Gaussian beam.

1. Introduction

The interaction of the incident shaped beam with non-spherical scatterer is important in numerous areas of applications, such as particle size measurement, Raman scattering diagnostics, optical levitation, and aerosol detection. The model of spheroidal particle is convenient to describe the non-spherical particles with finite sizes. Note that the spheroid has the practical value of approximating many slightly deformed particles to first order, e.g., raining drops [1], red blood cell [2], droplets subjected to aerodynamic drag [3], radiophysics spheroidal antennas, and dust grains in the interplanetary and interstellar medium. Therefore, the consideration of spheroids has the important applications in various fields of science.

During the years a lot of research efforts have been invested to the study of interactions between shaped beam and spheroidal particles. Based on the rigorous solution of Maxwell equations Asano and Yamamoto [4,5] studied the scattering properties of spheroidal particles illuminated by a plane wave. On the basis of their method, Voshchinnikov and Farafonov [6] improved the calculation efficiency by combining both Debye and Hertz potentials to describe the electromagnetic fields. Sebak and Sinha [7] presented a solution of electromagnetic plane wave scattering by a conducting spheroidal object with a dielectric coating, in which the conducting spheroidal object and the dielectric coating are concentric and confocal. The electromagnetic radiation from a prolate spheroidal antenna enclosed in a confocal radome was analyzed by Li et al. [8]. The geometrical optics approximation (GOA) is just appropriate to treat such a problem. Lock has studied the diffraction and specular reflection [9] as well as the transmission and cross-polarization effects [10] for an arbitrarily oriented spheroid in the case of plane wave illumination. Barton [11,12] calculated the internal and the near-surface electromagnetic fields for a spheroidal particle with arbitrary illumination, but he did not give the expressions for the

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beam shape coefficients (BSCs) of the Gaussian beam in spheroidal coordinates. Schulz et al. [13] used the T-matrix of spheroidal particles illuminated by a plane wave. Some numerical methods, such as Fourier Lorenz–Mie theory, discrete dipole approximation (DDA), etc., have been used to study scattering of Gaussian beam by particles [14–16].

A strong effort has been devoted during the two last decades to develop a set of theories, assembled under the common denomination of the Generalized Lorenz–Mie theory (GLMT), rigorously solving the problem of interaction between an arbitrary shaped beam and spheroid. The first paper concerning a GLMT for spheroidal particles is due to Han and Wu [17,18], who introduce an extrinsic method to evaluate the BSCs in spheroidal coordinates, namely they evaluate the spheroidal BSCs in terms of spherical BSCs. Subsequently, Han et al. [19] employed the GLMT to study the absorption and scattering of a Gaussian beam and a plane wave by an oblate particle for the on-axis case. In one previous paper, we achieved such an expansion of the incident arbitrary shaped beam in spheroidal coordinates in the general case of oblique illumination [20]. Based on our previous works, the purpose of this paper is to study the electromagnetic scattering properties of a spheroidal particle from an obliquely incident shaped beam. A focused Gaussian beam is taking into consideration due to its wide applications in researches, with the expectation that the numerical results given here would contribute to the understanding of focused beam scattering by non-spherical particles.

The paper is organized as follows. Section 2 provides the theoretical procedure for the determination of the scattered field of a Gaussian beam by a spheroid in oblique illumination. In Section 3, numerical results of Gaussian beam scattering properties for a spheroid are presented. Section 4 is a conclusion.

2. Theoretical treatment

The geometry of the specific scattering problem under study is illustrated in Fig. 1. The Gaussian beam propagates in free space and from the negative z' to the positive z' of $Ox'y'z'$, with the middle of beam waist located at origin O.

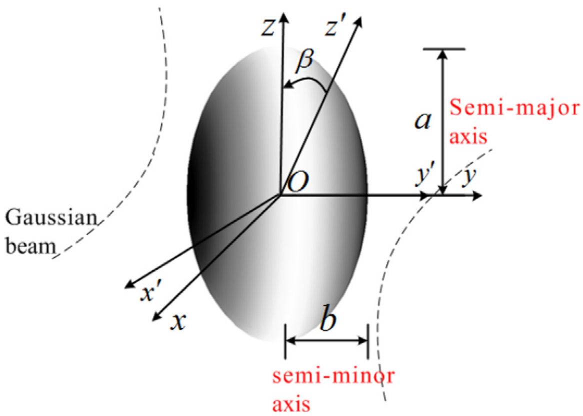

Fig. 1. Geometry for a Gaussian beam obliquely incident upon a prolate spheroid.

The center of a prolate spheroidal particle is located at origin O of Cartesian coordinate system $Oxyz$ which is obtained by rotating the system $Ox'y'z'$ through Euler angles β as defined in Ref. [21]. The semi-focal distance, semi-major and semi-minor axes of the spheroid are denoted by f, a and b. In this paper, we assume that the major axis is along the z axis, and the time-dependent part of the electromagnetic fields is $\exp(-i\omega t)$. From the scatter geometry described in Fig. 1, the beam description of $(E_{x'}^{(i)}, E_{y'}^{(i)}, E_{z'}^{(i)})$ and $(H_{x'}^{(i)}, H_{y'}^{(i)}, H_{z'}^{(i)})$ in the beam coordinates $Ox'y'z'$ can be transformed to their counterparts $(E_{x}^{(i)}, E_{y}^{(i)}, E_{z}^{(i)})$ and $(H_{x}^{(i)}, H_{y}^{(i)}, H_{z}^{(i)})$ in the spheroidal particle coordinates system $Oxyz$ via the following formula:

$$\begin{pmatrix} E_{x'}^{(i)} \\ E_{y'}^{(i)} \\ E_{z'}^{(i)} \end{pmatrix} = \mathbf{A} \begin{pmatrix} E_{x}^{(i)} \\ E_{y}^{(i)} \\ E_{z}^{(i)} \end{pmatrix} \tag{1}$$

$$\begin{pmatrix} H_{x'}^{(i)} \\ H_{y'}^{(i)} \\ H_{z'}^{(i)} \end{pmatrix} = \mathbf{A} \begin{pmatrix} H_{x}^{(i)} \\ H_{y}^{(i)} \\ H_{z}^{(i)} \end{pmatrix} \tag{2}$$

where the transformation matrix is given by

$$\mathbf{A} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \tag{3}$$

Within the framework of the GLMT, the electromagnetic fields of the incident Gaussian beam can be expanded in terms of the spheroidal vector wave functions $\mathbf{M}_{\substack{e\\o}mn}^{r(1)}(c,\zeta,\eta,\varphi)$ and $\mathbf{N}_{\substack{e\\o}mn}^{r(1)}(c,\zeta,\eta,\varphi)$ with respect to the system $Oxyz$ for the TE mode, as an example, in the following form [20]:

$$\mathbf{E}^i = E_0 \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n [G_{n,TE}^m \mathbf{M}_{emn}^{r(1)}(c,\zeta,\eta,\phi) + iG_{n,TM}^m \mathbf{N}_{omn}^{r(1)}(c,\zeta,\eta,\phi)] \tag{4}$$

$$\mathbf{H}^i = E_0 \frac{k}{\omega\mu} \sum_{m=0}^{\infty}\sum_{n=m}^{\infty} i^n [G_{n,TE}^m \mathbf{M}_{omn}^{r(1)}(c,\zeta,\eta,\phi) - iG_{n,TM}^m \mathbf{N}_{emn}^{r(1)}(c,\zeta,\eta,\phi)] \tag{5}$$

where $c=kf$, k is the wave-number in the surrounding medium, E_0 is the amplitude of the incident wave, and the expansion coefficients or spheroidal BSCs $G_{n,TE}^m$ and $G_{n,TM}^m$ can be expressed explicitly as

$$\begin{bmatrix} G_{n,TE}^m \\ G_{n,TM}^m \end{bmatrix} = -(-1)^{m-1} \frac{2}{N_{mn}} \sum_{r=0,1}^{\infty} {}' \frac{d_r^{mn}(c)}{(r+m)(r+m+1)} g_{r+m} \begin{bmatrix} (2-\delta_{0m}) \frac{dP_{r+m}^m(\cos\beta)}{d\beta} \\ 2m \frac{P_{r+m}^m(\cos\beta)}{\sin\beta} \end{bmatrix} \tag{6}$$

where $P_{r+m}^m(\cos\beta)$ designates the associated Legendre functions and $d_r^{mn}(c)$ are the expansion coefficients [17,22]. N_{mn} is