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# Scattering of Bessel beam by a conducting spheroidal particle with dielectric coating

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ABSTRACT

Based on the generalized Lorenz–Mie theory, an analytic solution to the scattering of an on-axis incident Bessel beam by a conducting spheroidal particle with dielectric coating is presented by expanding the incident beam, scattered fields and the fields in the dielectric coating in terms of spheroidal vector wave functions. In particular, the incident beam is represented using the vector expressions of zero-order Bessel beam that well satisfy Maxwell's equations. The unknown expansion coefficients for the scattered fields are determined by a system of linear equations derived from the appropriate boundary conditions. Numerical results of the differential scattering cross section are evaluated, and the scattering characteristics are discussed in detail.



## 1. Introduction

Over the last few decades, many researchers have investigated the scattering of electromagnetic wave by a spheroidal particle, due to the fact that many particles encountered in natural or produced in industrial process can be modeled by spheroids. Asano and Yamamoto [1] first studied the scattering of plane wave by a homogeneous spheroid by separating variables for the vector wave functions in the spheroidal coordinate system. Later, Sebak and Sinha [2] investigated the scattering of plane wave by a conducting spheroid with a confocal dielectric coating. For the scattering of an incident shaped beam by a spheroidal particle, an early study was carried out by Barton [3]. Using a spheroidal coordinate separation-of-variables solution, he calculated the internal and the near-surface fields for an arbitrary monochromatic field that is incident upon a spheroidal particle. Subsequently, within the framework of the generalized Lorenz–Mie theory (GLMT) developed by Gouesbet et al. [4,5], Han et al. [6,7] studied the scattering of an on-axis and off-axis Gaussian beam by a spheroidal particle. Zhang and Han [8] constructed an analytic solution to the scattering of Gaussian beam by a confocal multilayered spheroid. Sun et al. [9] investigated the scattering of Gaussian beam by a conducting spheroid with confocal dielectric coating. Zhang et al. [10] further analyzed the scattering of Gaussian beam by a conducting spheroid with a non-confocal dielectric coating. Nevertheless, to the best of our knowledge, the scattering of Bessel beam by a conducting spheroidal particle with dielectric coating has not been reported. In fact, the Bessel beam [11,12], as another kind of laser beam, has attracted the attention of many researchers because of its special characteristics of non-diffraction and self-reconstruction. And the scattering of Bessel beam from a sphere and experimental generation of Bessel beam have been studied in some literature, see [13–19] to quote of them. The purpose of this paper is to study the scattering of Bessel beam by a spheroid with dielectric coating by using the GLMT.

This paper is organized as follows. Section 2 presents the theoretical procedure for the scattering of Bessel beam by a conducting spheroid with dielectric coating. In Section 3,

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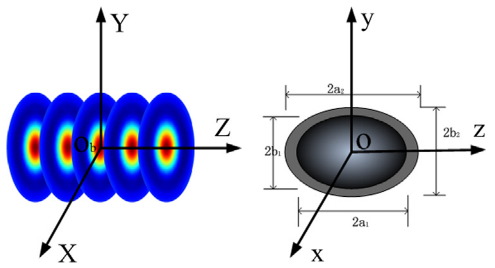


**Fig. 1.** Geometry for scattering of a Bessel beam by a conducting spheroidal particle with dielectric coating.

some numerical results are presented and discussed in detail. Section 4 is a conclusion.

## 2. Theory analysis

As shown in Fig. 1, we consider the scattering of a zero-order Bessel beam by a conducting spheroidal particle with dielectric coating. Two Cartesian coordinates *OXYZ* and *oxyz* are constructed, which are called beam coordinate system and particle coordinate system, respectively. Specially, the spherical coordinate is signed as $OR\theta\phi$. The incident beam propagates in free space from negative $Z$ to positive $Z$, which is attached to beam coordinate. In addition, the center of beam is fixed at the origin of beam coordinate. Furthermore, the conducting spheroidal particle with dielectric coating is attached to particle coordinate and its center is fixed at the origin of particle coordinate. The relationship of two coordinate systems can be expressed as

$$x = X, \quad y = Y, \quad z = Z - z_0. \tag{1}$$

The spheroid can be reformed by rotation of two axis of ellipse, and the rotation of the major axis and minor axis reform the prolate spheroid and oblate spheroid, respectively. In [6], Han et al. gave explicit definition of spheroid. As shown in Fig. 1, a prolate spheroid is studied in this paper. The common semifocal distance, the semimajor axis and the semiminor axis are denoted by $f$, $a$ and $b$, respectively. The right subscripts 1 and 2 are used to represent responding parameters of conducting spheroid and the coating spheroid, respectively. The inner boundary is set as $\zeta_1$, and the outer boundary is set as $\zeta_2$. The refractive index of the coating dielectric is $n$ relative to the free space. The wavenumber in free space and coating dielectric are denoted by $k$ and $k_1$, respectively.

In the following, the exacting theory of scattering of Bessel beam by a conducting spheroid with dielectric coating is given based on the GLMT [4]. In the GLMT, the vector expansion of electric and magnetic fields using the spherical vector wave functions is undoubtedly a key point, and the using of an appropriate vector expression of the incident beam is very important for vector expansion. In 1991, Mishra [20] derived the vector components for the electric and magnetic fields of a zero-order Bessel beam stemming from the Maxwell's vector equations and the Lorenz gauge condition. For the purpose of our theory analysis, it is reasonable to use this vector expression. The expressions are given as

$$E_X = \frac{1}{2} E_0 \left[ \left( 1 + \frac{k_Z}{k} - \frac{k_r^2 X^2}{k^2 r^2} \right) J_0(k_r r) - \frac{k_r (Y^2 - X^2)}{k^2 r^3} J_1(k_r r) \right] e^{ik_Z Z}, \tag{2}$$

$$E_Y = \frac{1}{2} E_0 XY \left[ \frac{2k_r}{k^2 r^3} J_1(k_r r) - \frac{k_r^2}{k^2 r^2} J_0(k_r r) \right] e^{ik_Z Z}, \tag{3}$$

$$E_Z = \frac{1}{2i} E_0 \frac{X}{kr} \left( 1 + \frac{k_Z}{k} \right) k_r J_1(k_r r) e^{ik_Z Z}, \tag{4}$$

$$H_X = \frac{\sqrt{\varepsilon}}{2} E_0 XY \left[ \frac{2k_r}{k^2 r^3} J_1(k_r r) - \frac{k_r^2}{k^2 r^2} J_0(k_r r) \right] e^{ik_Z Z}, \tag{5}$$

$$H_Y = \frac{\sqrt{\varepsilon}}{2} E_0 \left[ \left( 1 + \frac{k_Z}{k} - \frac{k_r^2 Y^2}{k^2 r^2} \right) J_0(k_r r) - \frac{k_r (X^2 - Y^2)}{k^2 r^3} J_1(k_r r) \right] e^{ik_Z Z}, \tag{6}$$

$$H_Z = \frac{\sqrt{\varepsilon}}{2i} E_0 \frac{Y}{kr} \left( 1 + \frac{k_Z}{k} \right) k_r J_1(k_r r) e^{ik_Z Z}, \tag{7}$$

where $E_0$ is the amplitude of electric field, $\varepsilon$ is the permittivity, $k_r = k \sin\alpha$ and $k_z = k \cos\alpha$ are the radial wave number and longitudinal wave number, respectively, with $k$ and $\alpha$ are the wave number in free space and the half-angle of the Bessel beam in background medium, respectively. Also, $r = \sqrt{X^2 + Y^2}$ is the radial distance to a point in transverse plane $(X,Y)$. Moreover, $J_{0,1}(k_r r)$ is the Bessel function of the first kind of the zeroth and first order, respectively. It is noted that the assuming time-dependent part of electromagnetic field $e^{-iwt}$ is given up for briefly.

The electric field of the incident on-axis Bessel beam can be represented in terms of spherical vector wave functions as

$$\vec{E}^i = \sum_{n=1}^{\infty} E_n g_n [\vec{M}^{(1)}_{o1n}(kR,\theta,\phi) - i\vec{N}^{(1)}_{e1n}(kR,\theta,\phi)], \tag{8}$$

and spherical vector wave functions $\vec{M}^{(1)}_{omn}$ and $\vec{N}^{(1)}_{emn}$ can be expressed as [21]

$$\vec{M}^{(1)}_{o1n}(r,\theta,\phi) = \frac{1}{\sin\theta} j_n(kr) P_n^1(\cos\theta)\cos\phi\,\hat{e}_\theta - j_n(kr)\frac{dP_n^1(\cos\theta)}{d\theta}\sin\phi\,\hat{e}_\phi, \tag{9}$$

$$\vec{N}^{(1)}_{e1n}(r,\theta,\phi) = \frac{n(n+1)}{kr} j_n(kr) P_n^1(\cos\theta)\cos\phi\,\hat{e}_r + \frac{1}{kr}\frac{d}{d(kr)}[krj_n(kr)]\frac{dP_n^1(\cos\theta)}{d\theta}\cos\phi\,\hat{e}_\theta - \frac{1}{kr\sin\theta}\frac{d}{d(kr)}[krj_n(kr)]P_n^1(\cos\theta)\sin\phi\,\hat{e}_\phi, \tag{10}$$

where $P_n^1(\cos\theta)$ is the associated Legendre function and $j_n(kr)$ is the spherical Bessel function.

With reference to the results of expansion coefficients in our previous investigation about the scattering of Bessel beam by a concentric sphere [22], the expression of $E_n$ and