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Scattering by multiple cylinders buried in a lossy half space at oblique incidence

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ABSTRACT

The scattering characteristics of an obstacle are a function of its size, shape, and refractive index, as well as the refractive index of the host medium. For two-dimensional scatterer such as an infinite cylinder, they are also dependent on the incidence direction. The scattering problem is two-dimensional if the incident wave propagates perpendicular to the axis of the cylinder, whereas the problem becomes three-dimensional when the incident direction is inclined from the cylinder axis. The latter case of oblique incidence at a lossy half space containing multiple infinite cylinders is treated in this paper. The theoretical treatment utilizes Hertz potentials to formulate the propagation of inhomogeneous waves in the lossy medium, reflection of depolarized scattered waves at the half space interface, and transmission of scattered waves from within the half space. Analytical formulas are derived for the electromagnetic fields and Poynting vector of the scattered radiation emerging from the half space. Numerical examples are presented to illustrate scattering by multiple dissimilar infinite cylinders buried in a lossy half space irradiated by an obliquely incident plane wave.

1. Introduction

Detection and diagnosis of cylindrical objects buried in a bounded host medium is relevant to many problems such as imaging of biological tissues, light scattering in photonic crystals, detection of subterranean pipes and conduits, etc. Despite the dissimilar scales in the various applications, they involve the same fundamental principles because the wavelength of the source radiation would be comparable to the diameter of the cylindrical obstacles. The region of illumination is usually large compared to the length and diameter of the cylinders, so that they can be treated as infinitely long. The approximation of infinite cylinder allows exact analytical treatment of many scattering problems that involve buried cylindrical obstacles.

Many studies have been reported on scattering by a single or multiple infinite cylinders buried in a non-absorbing (lossless) [1–7] and absorbing (lossy) [8–15] half space. The half space interface causes reflection of scattered waves, which become secondary incident waves on the cylinders. When the host medium is lossy, the scattered waves traversing the medium become inhomogeneous waves that decay with distance. Because an infinite cylinder is two-dimensional, its scattering characteristics are dependent on the direction of the incident wave. At perpendicular incidence, i.e. the incident wave propagates in the plane normal to the axis of the cylinder, the scattered waves propagate in the same plane and its polarization would remain unchanged from that of the incident wave. Oblique incidence occurs if the incident wave propagates outside of this normal plane, so that the incident direction is inclined from the cylinder axis. The scattered waves would become depolarized and propagate into both polar and azimuth directions. The depolarized scattered waves would be reflected at the boundary and become incident waves on the cylinders. Further depolarization of the scatter-incident waves ensues because they are at oblique incidence on the cylinders. As a result the problem becomes considerably more complicated due to the 3-dimensional scattering

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characteristics and the cascading effect of reflection of the depolarized scattered waves at the interface.

Most of the published literature are restricted to perpendicular incidence, except for Lee and Grzesik [4] who treated oblique incidence for a non-absorbing host medium. As the orientation of the buried cylindrical obstacles is generally not known a priori in practical situations, the probing radiation is more likely to deviate from perpendicular incidence. No theoretical study can be found in the literature for oblique incidence on cylindrical obstacles buried in a lossy substrate. A scattering solution for arbitrary incident direction on multiple cylinders buried in a lossy half space is needed for the analysis and interpretation of backscattering data in applicable remote sensing problems.

The objective of this paper is to present a theoretical solution for oblique incidence on closely spaced parallel infinite cylinders buried in a lossy half space. Each cylinder is distinct and radially stratified, and the incident wave is arbitrarily polarized. The theoretical treatment utilizes Hertz potentials to formulate the inhomogeneous waves in the lossy medium, near-field interactions between cylinders, and reflection and transmission of scattered waves at the interface. Surface roughness is excluded in the present consideration, so that the interface is perfectly smooth and the Fresnel law applies. The theoretical formulation is detailed in the following sections, followed by numerical results to illustrate application of the solution.

2. Theory

The geometry of the present problem is depicted in Fig. 1a and b. An arbitrarily polarized plane wave propagates in medium 1 in the direction (θ_1, ϕ_1) relative to the reference frame $OXYZ$. The azimuth angle θ_1 is measured in the XY plane and ϕ_1 is the polar angle inclined from the XZ plane. Medium 1 is a lossless dielectric with real refractive index $\tilde{m}_1$, permeability μ_1, and propagation constant $k_1(=\tilde{m}_1 k_o)$, where k_o is the free space wavenumber. Medium 2 is lossy with complex properties $(\tilde{m}_2, \mu_2, k_2 = \tilde{m}_2 k_o)$ in which multiple infinite cylinders parallel to the Z-axis are located. Each cylinder is radially stratified with complex properties $(\tilde{m}_{jq}, \mu_{jq}, k_{jq}, q = 1{:}\,Q_j)$, where Q_j is the number of radial layers in cylinder j.

The source radiation transmitted across the medium 1–2 interface is the primary incident wave on the cylinders. Near-field scattered waves from the cylinders are secondary incident waves, while the scattered waves reflected at the half space interface are denoted as tertiary incident waves. The total electric (E) and magnetic (H) fields in the vicinity of a cylinder consists of contributions from all these sources as

$$\begin{pmatrix} \vec{E}_{\psi j} \\ \vec{H}_{\psi j} \end{pmatrix} = \begin{pmatrix} \vec{E}^{+}_{\psi j} \\ \vec{H}^{+}_{\psi j} \end{pmatrix} + \begin{pmatrix} \vec{E}^{s}_{\psi j} \\ \vec{H}^{s}_{\psi j} \end{pmatrix} + \sum_{k \neq j}^{N} \begin{pmatrix} \vec{E}^{s}_{\psi,kj} \\ \vec{H}^{s}_{\psi,kj} \end{pmatrix} + \sum_{k=1}^{N} \begin{pmatrix} \vec{E}^{r+}_{\psi,kj} \\ \vec{H}^{r+}_{\psi,kj} \end{pmatrix}. \tag{1}$$

The effects of multiple cylinders and the interface can be seen in the terms on the right hand side. For a single cylinder in an unbound medium, the total field would contain only the primary incident wave and scattered wave that correspond to the first two terms, respectively. The presence of multiple cylinders causes near-field interaction between cylinders, which gives rise to the 3rd term. The last term arises from the reflection of scattered waves at the interface. These fields satisfy the equations

$$\vec{E} = \nabla \times (\vec{e}_z v) + \frac{i}{k} \nabla \times \nabla \times (\vec{e}_z u). \tag{2}$$

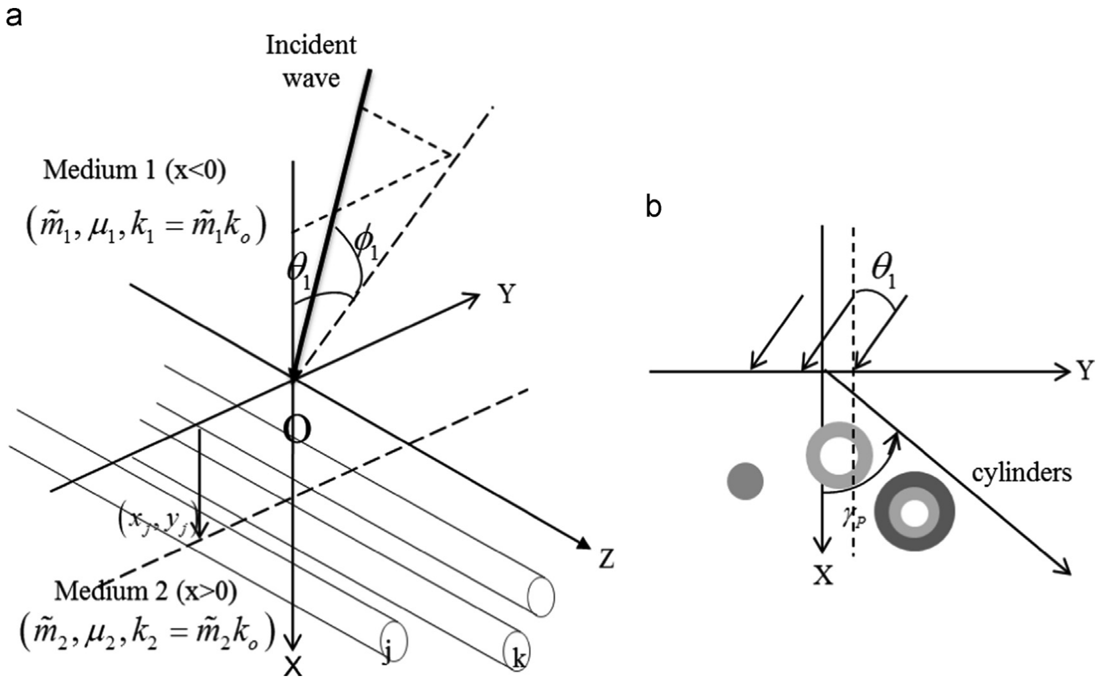

Fig. 1. Schematic diagram of the present problem.