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Electromagnetic scattering by a partially charged multilayered sphere

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ABSTRACT

A new calculation procedure for the attenuation coefficients of electromagnetic wave by a partially charged multilayered sphere is proposed. The procedure is based on the utilization of a prescription which relates the expansion coefficients of the electromagnetic fields in the n-layered zone to those for the core zone through an iterative process, and then directly applies the coated-sphere model to calculate the expansion coefficients of the scattering field, and the extinction cross section. This method can be used to calculate the scattering properties of any multilayer charged sphere.

1. Introduction

Electromagnetic scattering from the particulate matter is an important research subject in astronomy, astrophysics, biomedicine, polymer science, atmospheric science, and laboratory experiments [1]. We usually use the Mie method [2], T-matrix method [3–6], DDA [7], or Sh-matrix [8,9] to predict the scattering and absorption of light by a homogeneous particle. However, in real environment, the particle is made of inhomogeneous medium [10], for example, the particle may have a core-shell structure [11], or be multilayered [12–14]. In order to calculate the scattering properties of these kinds of heterogeneous particles, some researchers developed various methods, for example, the n-layered Mie solution [15], Debye series [16–19], equivalent dielectric constant [20], and so on. Some papers also discussed the improved recursive algorithm for the light scattering by a multilayered sphere [15,21]. Although the previous methods mentioned above are much simpler and more accurate in estimating the particle's optical properties, they are options available for calculating the scattering of the uncharged particle.

Actually, some common granules in nature or in industry are charged when they are moving [22,23]. For example, the sands in wind-blown sand layers or in the sandstorms are charged, and the charge can reach up to 1120.7 μC/kg [24]. There are several reasons why the sands are charged when they come in contact with each other [25], some papers have shown that the charged ion or electron on particle surface was anisometrically transferred when the particles came in contact, hence making them charged when separated [25,26], which means the charge may only be distributed on the particle surface. The charge on the particle not only produces a strong electric field, but also enhances the scattering and absorption of

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the incident electromagnetic wave by particles [27–32]. However, there are no reports on the scattering properties of charged multilayered particles.

In this paper, we want to propose a simple method to predict the optical properties of a partially charged multilayered particle. The procedure is that we first found an iteration relation for the expansion coefficients of electromagnetic fields between the last layered zone and the core zone, and then directly used the charged coated-sphere model which was proposed by Li et al. [33] to obtain the expansion coefficients of the scattering field, finally acquired the extinction cross section for any charged multilayer spherical particle.

2. Scattering of charged multilayered spherical particle

For simplicity, suppose a plane x-polarized wave (the wavelength is λ) illuminates a small partially charged multilayer particle, and there is no charge on the interface between any two shells. For the core, its radius is R_0 and the permittivity is ε_o. For the *j-layer* shell ($j=1,2\cdots N$), suppose its radius is R_i, and the permittivity is ε_i. The background medium is air and its permittivity is ε_m,. The charge uniformly distributes on a spherical cap of the particle surface with an angle of $2\theta_0$, as shown in Fig. 1, and we just show a cross section at $\varphi=0$. As shown by some references [15,33], the incident field $\boldsymbol{E}_{in}$, $\boldsymbol{H}_{in}$, the interior field in core zone $\boldsymbol{E}_0$, $\boldsymbol{H}_0$ the interior field in the *i-layer* shell zone $\boldsymbol{E}_i$, $\boldsymbol{H}_i$,, the scattering field $\boldsymbol{E}_s$, $\boldsymbol{H}_s$ all can be expanded in vector spherical harmonics functions,

$$\boldsymbol{E}_{in}=\sum_n E_n\left[\boldsymbol{M}^1_{o1n}(\rho,\theta,\varphi)-i\boldsymbol{N}^1_{e1n}(\rho,\theta,\varphi)\right]$$

$$\boldsymbol{H}_{in}=-\frac{k}{\omega\mu}\sum_n E_n\left[\boldsymbol{M}^1_{e1n}(\rho,\theta,\varphi)+i\boldsymbol{N}^1_{o1n}(\rho,\theta,\varphi)\right] \tag{1a}$$

$$\boldsymbol{E}_{s}=\sum_n E_n\left[ia_n\boldsymbol{N}^3_{e1n}(\rho,\theta,\varphi)-b_n\boldsymbol{M}^3_{o1n}(\rho,\theta,\varphi)\right]$$

$$\boldsymbol{H}_{s}=-\frac{k}{\omega\mu}\sum_n E_n\left[ib_n\boldsymbol{N}^3_{o1n}(\rho,\theta,\varphi)+a_n\boldsymbol{M}^3_{e1n}(\rho,\theta,\varphi)\right] \tag{1b}$$

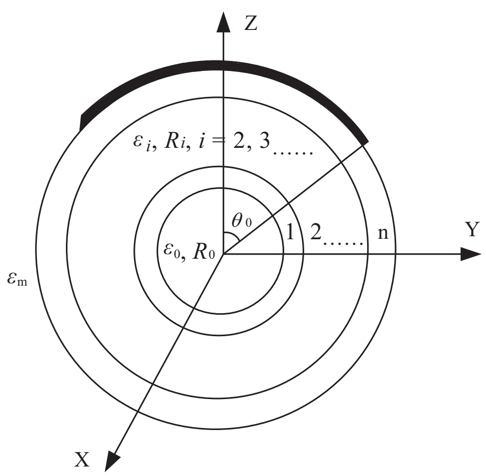

Fig. 1. The physical model for a multiplayer spherical particle.

$$\boldsymbol{E}_{0}=\sum_n E_n\left[c^{(0)}_n\boldsymbol{M}^1_{o1n}(\rho_0,\theta,\varphi)-id^{(0)}_n\boldsymbol{N}^1_{e1n}(\rho_0,\theta,\varphi)\right]$$

$$\boldsymbol{H}_{0}=-\frac{k_0}{\omega\mu_0}\sum_n E_n\left[d^{(0)}_n\boldsymbol{M}^1_{e1n}(\rho_0,\theta,\varphi)+ic^{(0)}_n\boldsymbol{N}^1_{o1n}(\rho_0,\theta,\varphi)\right] \tag{1c}$$

$$\begin{aligned}\boldsymbol{E}_{j}=\sum_n E_n\Big[&c^{(j)}_n\boldsymbol{M}^1_{o1n}(\rho_j,\theta,\varphi)-id^{(j)}_n\boldsymbol{N}^1_{e1n}(\rho_j,\theta,\varphi)\\&+f^{(j)}_n\boldsymbol{M}^2_{o1n}(\rho_j,\theta,\varphi)-ig^{(j)}_n\boldsymbol{N}^2_{e1n}(\rho_j,\theta,\varphi)\Big]\\ \boldsymbol{H}_{j}=-\frac{k_j}{\omega\mu_j}\sum_n E_n\Big[&d^{(j)}_n\boldsymbol{M}^1_{e1n}(\rho_j,\theta,\varphi)+ic^{(j)}_n\boldsymbol{N}^1_{o1n}(\rho_j,\theta,\varphi)\\&+g^{(j)}_n\boldsymbol{M}^2_{e1n}(\rho_j,\theta,\varphi)+if^{(j)}_n\boldsymbol{N}^2_{o1n}(\rho_j,\theta,\varphi)\Big]\end{aligned} \tag{1d}$$

Here $\rho=kr, k=2\pi/\lambda, \rho_j=m_jx_j=\sqrt{\varepsilon_j/\varepsilon_m}x_j, x_j=kR_j, k_0=\ m_0k=\sqrt{\varepsilon_0/\varepsilon_m}k, k_j=m_jk, \mu, \mu_0$ *and* μ_j are the permeability of air, the core zone and the j-th shell, respectively. In this paper we do not consider the permeability of the sands, so $\mu=\mu_0=\mu_j=4\pi\times10^{-7}\ N/m^2.a_n, b_n, c^{(0)}_n, d^{(0)}_n, c^{(j)}_n, f^{(j)}_n, d^{(j)}_n, g^{(j)}_n$ is the expansion coefficient of the electromagnetic field for the zone inside or outside the particle, $j=1,2\cdots N$. $\boldsymbol{M}^j_{e1n}, \boldsymbol{M}^j_{o1n}, \boldsymbol{N}^j_{e1n}$ and $\boldsymbol{N}^j_{o1n}$ are the vector spherical harmonics functions, which must meet the following equations:

$$\boldsymbol{M}_{en}=\nabla\times(\boldsymbol{r}\psi_{en}), \boldsymbol{M}_{on}=\nabla\times(\boldsymbol{r}\psi_{on}), k\boldsymbol{N}_{en}=\nabla\times\boldsymbol{M}_{en}, k\boldsymbol{N}_{on}=\nabla\times\boldsymbol{M}_{on},$$

$$\begin{aligned}&\psi^{(1)}_{on}=\ \sin\,\varphi P^{(1)}_n(\cos\,\theta)j_n(kr) \quad \psi^{(1)}_{en}=\ \cos\,\varphi P^{(1)}_n(\cos\,\theta)j_n(kr)\\&\psi^{(2)}_{on}=\ \sin\,\varphi P^{(1)}_n(\cos\,\theta)y_n(kr) \quad \psi^{(2)}_{en}=\ \cos\,\varphi P^{(1)}_n(\cos\,\theta)y_n(kr)\\&\psi^{(3)}_{on}=\ \sin\,\varphi P^{(1)}_n(\cos\,\theta)h_n(kr) \quad \psi^{(3)}_{en}=\ \cos\,\varphi P^{(1)}_n(\cos\,\theta)h_n(kr)\end{aligned} \tag{2}$$

Here $P^{(m)}_n(\cos\,\theta)$ is the first associated Legendre function but $m=1$, $j_n(x)$ and $y_n(x)$ are the spherical Bessel functions of the first and second kinds and $h_n(x)=j_n(x)+iy_n(x)$ is the spherical Bessel function of the third kind, respectively.

To gain the expansion coefficients for the interior field and the scattering field, we must use the boundary conditions, which can be represented as follows [30,32,33]:

$$r=R_N:\quad \mathbf{n}\times(\boldsymbol{E}_{in}+\boldsymbol{E}_s-\boldsymbol{E}_N)=0 \tag{3a}$$

$$r=R_N:\quad \mathbf{n}\times(\boldsymbol{H}_{in}+\boldsymbol{H}_s-\boldsymbol{H}_N)=\boldsymbol{K}H(\theta-\theta_0) \tag{3b}$$

$$r=R_0:\quad \boldsymbol{n}\times\boldsymbol{E}_0=\boldsymbol{n}\times\boldsymbol{E}_1 \tag{3c}$$

$$r=R_0:\quad \boldsymbol{n}\times\boldsymbol{H}_0=\boldsymbol{n}\times\boldsymbol{H}_1 \tag{3d}$$

$$r=R_j:\quad \boldsymbol{n}\times\boldsymbol{E}_j=\boldsymbol{n}\times\boldsymbol{E}_{j-1} \tag{3e}$$

$$r=R_j:\quad \boldsymbol{n}\times\boldsymbol{H}_j=\boldsymbol{n}\times\boldsymbol{H}_{j-1} \tag{3f}$$

Here $\boldsymbol{K}=\sigma_s\boldsymbol{E}_{Nt}$ is the surface current density, $H_{(\theta-\theta_0)}$ is the Heaviside function, $\boldsymbol{E}_{Nt}$ is the tangential component of the surface electric field, and

$$\sigma_s=\frac{i\sigma q_e}{m_e(\omega+i\sigma\pi r_e^2\sqrt{T/m_e})} \tag{4}$$

is the surface conductivity [32], m_e, q_e, r_e are the mass, charge, and radius of the electron, σ is the charge density on particle surface, ω is the angular frequency of the incident wave and T is the surface temperature. Substituting those expansions (1a–1d) for the incident field, the