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Gray radiation transport models for two-dimensional binary stochastic media with material temperature coupling using spherical harmonics

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ABSTRACT

Due to the high computational cost of time-dependent radiation transport calculations, most multi-dimensional simulations of stochastic media have used the lowest angle order approximation, the P_1 approximation. Here spherical harmonics of order $n=5$ are used to solve the transport equation in two-dimensional binary stochastic media. The results are consistent with earlier P_1 simulations. Transport solutions using constant and temperature-dependent opacities and heat capacities are shown and analyzed. The standard closure poorly approximates the mean radiation field in these test problems. For one physical case, a less common closure is better. To best fit the most general cases, a new procedure is presented. In all cases, the approximate transport solutions work best in dilute systems where one material comprises less than about 10% of the total. The conclusions reached here should be independent of the transport solution method whether one uses spherical harmonics or discrete ordinates.

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1. Introduction

There are many situations where there is interest in the transport of radiation through stochastic media. Raindrops in clouds are complex because electrical charges may cause them to be not totally random. Molecular clouds in the interstellar medium provide lumps of opacity to hinder the flow of starlight. Nuclear reactors can use regular rod structures or randomly placed pellets. Turbulent flows in inertial confinement fusion plasmas can look locally random with larger scale gradients. The test cases presented here are generic. The geometry and physical attributes are not intended to model any specific situation. The goals of this work are to increase the understanding radiation flow through multi-dimensional stochastic media and to find approximate solutions for the mean radiation field.

Many authors have published studies of radiation transport through binary stochastic media with the purpose of trying to find a model that approximates the ensemble averaged transport solution with easy to solve equations. Early work (for example [1–4]) treated pure radiation with no material coupling. Then research began to include coupling with a material equation [5,6]. All this work was done in one-dimensional slab or rod geometries. Unfortunately, it was found in [7] that models such as the standard closure of Levermore and Pomraning (LP) [1] that can be successful in one dimension do not work as well in multidimensions.

Previous work by this author [7,8] in multidimensions used the lowest angle-order approximation, P_1, in order to reduce the computational cost of the transport solutions. In the present work, spherical harmonics [9,10] of order $n=5$ are used for the transport solutions. This is a high

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enough order to capture most of the angle dependence of the test problems and is a low enough order to be computationally affordable on a personal computer. The conclusions arrived at here are consistent with the earlier conclusions based on the $n=1$ solutions. No fundamental changes are observed when using higher-order transport solutions.

Simplifying assumptions are often made in order to make theoretical analysis more tractable. Pure radiation with no material coupling is obviously much simpler. When coupled to a material energy balance equation, it is often assumed that the opacities are constant and the heat capacity varies as the cube of the material temperature, T. This allows for a linearization of the equations around the material energy variable, $B=T^4$. Unfortunately, no known material has such a temperature dependent heat capacity. A constant heat capacity is more physically reasonable. Therefore, since it is not relevant to real-world transport, linearized transport will not be examined here. Three test problems are presented: (a) constant opacities and constant heat capacities, (b) opacities varying inversely with the temperature and constant heat capacities, and (c) opacities varying inversely with the temperature and a Saha-like temperature-dependent heat capacity.

Several methods for approximating the mean radiation field are tested. It is found that different methods may be optimum depending on what physics is assumed. The standard LP closure [1] does not work well on any of the test problems examined here. An alternate closure [4] fits all the cases reasonably well, but only with temperature independent opacities and heat capacities are the results useful in a predictive sense. A more generalized approach works well for the cases with temperature dependent physics, but does poorly on the test problem with constant opacities and heat capacities.

The following section presents model equations that attempt to calculate the mean radiation energy density in a two-material stochastic medium. Then three sections examine the results for the test problems. In each case a model is found that may be useful in calculations that do not resolve the stochastic nature of the medium. The last section gives a summary and some conclusions.

2. Equations used to model stochastic media

The standard closure in a binary medium uses two coupled transport equations, one for each material. For notational simplicity, the equations are presented here in one-dimensional slab geometry. As in previous work [7,8], the transport for material i can be written as

$$\frac{1}{c}\frac{\partial I_i}{\partial t}+\mu\frac{\partial I_i}{\partial z}=-\sigma_{ti}I_i+\frac{c}{4\pi}(\sigma_{ai}T_i^4+\sigma_{si}E_i)+\frac{\alpha}{\lambda_i}(I_j-I_i)+\frac{\beta}{4\pi\lambda_i}(E_j-E_i), \quad (1)$$

where the i subscript represents one material, 1 or 2, and j represents the other, 2 or 1, respectively. The first three terms on the right hand side are the usual absorption, emission, and scattering terms. The remaining terms couple the transport equations for the two materials. The scaling on these terms is a combination of an arbitrary dimensionless constant (α or β) and the mean chord length in each material, λ_i (defined in a following paragraph). The terms scaled by α are the standard closure used by most authors, are often attributed to [1], and are usually referred to as the Levermore and Pomraning (LP) closure. The terms scaled by β were proposed in [4], but have not been used widely by other authors. As originally proposed, these closures assume that $\alpha=1$ and $\beta=1$. Much better fits to the mean radiation field in the stochastic medium are found with values other than unity. When modeling a medium consisting of 1D alternating slabs, then there is an additional angle cosine factor multiplying the α term. When modeling a homogenized two- or three-dimensional medium, the cosine factor is not appropriate [6,8].

The material equation to be solved with Eq. (1) is given by

$$\frac{c_{Vi}}{c}\frac{\partial T_i}{\partial t}=\sigma_{ai}(E_i-T_i^4), \quad (2)$$

where c_{Vi} is the heat capacity and T_i is the temperature of material i. Here we are using units such that $B=T^4$ and $E=T_r^4$ are the material and radiation energy densities and T_r is the radiation temperature.

Consider a stochastic medium consisting of identical high opacity non-overlapping disks randomly placed in square 1 cm on a side. The disks are labeled as material 1 and have an absorption opacity of σ_{a1}. The probability that any point in this region lies inside such a disk is p_1. When disks intersect the boundary, only that part inside the unit square is included in p_1. The background low opacity material covers the remaining area with $p_2=1-p_1$. A mean chord length, λ_i, characterizes each material. It measures how much of a ray randomly placed in the medium is covered by each material. These quantities are related through the expression $p_i=\lambda_i/(\lambda_1+\lambda_2)$. If all the disks have the same radius, the mean chord length across them is $\pi/2$ times the radius of the disks (see [11]). This 2D Cartesian medium actually represents a 3D medium with infinitely long rods; however, it is easier and common in the literature to discuss it as a 2D medium with disks.

The goal of solving Eqs. (1) and (2) is to find the mean radiation field for an ensemble average of media with different disk locations. The mean intensity and temperature are simply given by $I=p_1I_1+p_2I_2$ and $T=p_1T_1+p_2T_2$. These predicted means are compared to the actual means obtained by averaging multiple realizations of the stochastic medium.

An analysis by Prinja and Olson [6] started with the above LP equations and found an asymptotic regime where these equations reduced to a single standard transport equation with effective coefficients coupled to a single material equation:

$$\frac{1}{c}\frac{\partial I}{\partial t}+\mu\frac{\partial I}{\partial z}=-\sigma_{t,eff}I+\frac{c}{4\pi}(\sigma_{a,eff}T^4+\sigma_{s,eff}E) \quad (3a)$$

$$\frac{c_V}{c}\frac{\partial T}{\partial t}=\sigma_{a,eff}(E-T^4). \quad (3b)$$

This process required the introduction of scaling parameters. These parameters and the resulting effective opacities for their high-contrast opacity analysis are

$$\langle\sigma_a\rangle=p_1\sigma_{a1}+p_2\sigma_{a2}, \quad \tilde{\sigma}=p_1\sigma_{a2}+p_2\sigma_{a1}, \quad \hat{\sigma}=\tilde{\sigma}+\frac{\alpha}{\lambda_c}, \quad (4a)$$