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Zero slopes of the scattering function and scattering matrix for strict forward and backward scattering by mirror symmetric collections of randomly oriented particles

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A B S T R A C T

Single scattering of light by a finite mirror symmetric collection of independently scattering randomly oriented particles is considered as observed in the far-field. It is shown that the slopes of the scattering function and all other elements of the scattering matrix are functions of the scattering angle that tend to zero when the direction of the scattered light tends to the strict forward or backward direction. This result is obtained by introducing an extended scattering matrix, based on symmetry arguments. The theory is illustrated and clarified by practical examples of scattering functions and scattering matrices. Various applications are also considered.

1. Introduction

In theoretical and experimental studies of light scattering by particles a key role is played by the scattering matrix. This 4×4 matrix determines the four Stokes parameters of singly scattered light traveling in a certain direction for a given beam of polarized incident light [1–4]. The scattering matrix depends in general on a polar angle, θ, in the closed range $[0, \pi]$, and an azimuth angle, ϕ, in the closed range $[0, 2\pi]$, where $\theta = 0$ for strict forward scattering and $\theta = \pi$ for strict backward scattering. In this paper we only consider single scattering by finite mirror symmetric collections of randomly oriented particles. The particles scatter light independently and a detector is located in the far-field. Such collections are frequently met in theoretical and numerical work on light scattering. In practice they are often very suitable approximations. Due to rotational symmetry of the collections there is no dependence on azimuth and the scattering matrix can be written as $\boldsymbol{F}(\theta)$.

The first element of $\boldsymbol{F}(\theta)$ is the scattering function $F(\theta)$. This scalar function is the only element we need when polarization is ignored. In general it has several interesting features like maxima and minima [1–5]. The features near strict forward and backward scattering are, however, difficult to uncover by experimental means [6]. Furthermore, in results of numerical computations the behavior of $F(\theta)$ when θ tends to zero or π is often not clearly shown, due to the use of an insufficient number of values for the scattering angle (i.e. the mess is too coarse). This happens in particular for the strong forward peak of the scattering function produced by large particles. Similar problems near strict forward and backward scattering occur for some other elements of the scattering matrix.

In this paper we study the behavior of all elements of the scattering matrix when the scattering angle tends to zero or π. In Section 2 the form of the scattering matrix, $\boldsymbol{F}(\theta)$, is discussed and the extended scattering matrix, $\boldsymbol{G}(\theta)$, is introduced. It is shown that all elements of $\boldsymbol{G}(\theta)$ have a

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horizontal tangent if θ is zero or π. For this reason the slopes of all elements of $\boldsymbol{F}(\theta)$ tend to zero when θ tends to zero or π. This also holds for a number of combinations of such elements, as proven in Section 3. The next section is devoted to examples that clarify and corroborate the theory expounded in the preceding sections. Various applications of the main results of the theory are discussed in Section 5. Appendix A is devoted to derivatives of electric fields, Stokes parameters and elements of the extended scattering matrix. In Appendix B some general properties of polynomials are employed to show that all generalized spherical functions have finite derivatives with respect to x and θ, where $x = \cos(\theta)$.

2. Scattering in a plane

Suppose a finite collection of independently scattering particles at the origin, O, of a Cartesian coordinate system is illuminated by a beam of light and provides beams of singly scattered light in all directions in three dimensional space. A detector is located at a far-field observation point. Fig. 1 shows the situation for a direction of the scattered light with polar angle θ. We use Stokes parameters I, Q, U and V to describe the intensity (or flux) and state of polarization of a beam of quasi-monochromatic light and make these parameters elements of a column vector, $\boldsymbol{I} = [I, Q, U, V]^t$, called the Stokes vector, where the superscript t stands for transpose [1,4]. Here I is positive and not smaller than the absolute value of any of the other Stokes parameters. The reference plane for the Stokes parameters is the plane of scattering, i.e. the plane defined by the directions of the incident and scattered light beams.

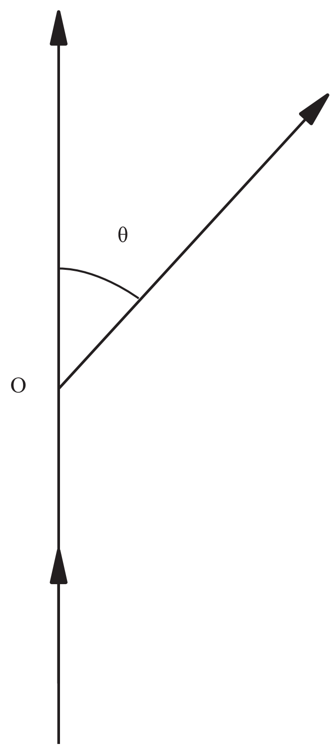

Fig. 1. Light scattering by a collection of particles at a point O in a direction making an angle θ with the direction of the incident light.

We can now write

$$\boldsymbol{I}^s(\theta) = c\boldsymbol{F}(\theta)\boldsymbol{I}^i(0), \tag{1}$$

where $\boldsymbol{I}^i(0)$ and $\boldsymbol{I}^s(\theta)$ are the Stokes vectors of the incident and scattered light beams, respectively, c is a positive constant that does not depend on θ and $\boldsymbol{F}(\theta)$ is the 4 by 4 scattering matrix. It is important to note that the angle θ in Eq. (1) is restricted to the range $0 \le \theta \le \pi$. For an arbitrary collection of particles the Stokes vector of the scattered light and the scattering matrix may not only depend on θ, but also on an azimuthal angle, as far as directions are concerned. However, this is not the case in this paper, since we only consider mirror symmetric collections of randomly oriented particles. Therefore, the scattering matrix is of the form:

$$\boldsymbol{F}(\theta) = \begin{pmatrix} F_{11}(\theta) & F_{12}(\theta) & 0 & 0 \\ F_{12}(\theta) & F_{22}(\theta) & 0 & 0 \\ 0 & 0 & F_{33}(\theta) & F_{34}(\theta) \\ 0 & 0 & -F_{34}(\theta) & F_{44}(\theta) \end{pmatrix}, \tag{2}$$

where $F_{ij}(\theta)$ stands for the element in the i-th row and j-th column of $\boldsymbol{F}(\theta)$. Among the collections included are [1] the following:

(i) randomly oriented particles with a plane of symmetry, like spheres, bi-spheres, spheroids, cylinders, cubes, etc.,
(ii) randomly oriented particles with their mirror particles in equal numbers, like right-handed screws and left-handed screws,
(iii) randomly oriented particles that are so small compared to the wavelength that Rayleigh scattering is sufficiently accurate, like molecules for visible incident light.

The positive element $F_{11}(\theta)$ is the scattering function and can also be written as $F(\theta)$. The absolute value of each other element is smaller than or equal to $F_{11}(\theta)$. The relations $F_{21}(\theta) = F_{12}(\theta)$ and $F_{43}(\theta) = -F_{34}(\theta)$ are due to reciprocity. The fact that the eight elements of the 2×2 matrices in the lower left and upper right corners are identically equal to zero is due to mirror symmetry with respect to the scattering plane. This was briefly mentioned by Hovenier in 1969 [7] and treated more extensively in [4].

To study the behavior of the scattering matrix when θ tends to zero or π we will now extend the range of θ by measuring θ clockwise from the forward scattering direction in the range $[0, 2\pi]$, which is equivalent to measuring θ anti-clockwise from the forward scattering direction in the range $[0, -2\pi]$. This is shown in Fig. 2 where we have $0 \le \theta \le \pi$ in S_1, i.e. the right half-plane (as in Fig. 1) and $-\pi \le \theta \le 0$ in S_2, i.e. the left half-plane. For strict forward scattering $\theta = 0$ and for strict backward scattering $\theta = \pm\pi$. The directions given by θ and $-\theta$ of the scattered beam are symmetric with respect to the strict forward as well as strict backward scattering directions. We have thus combined two half-planes into one complete plane. We can now write instead of Eq. (1):

$$\boldsymbol{I}^s(\theta) = c\boldsymbol{G}(\theta)\boldsymbol{I}^i(0), \tag{3}$$

where $-\pi \le \theta \le \pi$. We use $G_{ij}(\theta)$ to denote the element in the i-th row and j-th column of $\boldsymbol{G}(\theta)$. For a fixed beam of