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Analytical properties of the radiance in atmospheric radiative transfer theory

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A R T I C L E I N F O

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A B S T R A C T

It is demonstrated mathematically strictly that state density functions, as the radiance (specific intensity), exist to describe certain state properties of transported photons on microscopic and the state of the radiation field on macroscopic scale, which have independent physical meanings. Analytical properties as boundedness, continuity, differentiability and integrability of these functions to describe the photon transport are discussed. It is shown that the density functions may be derived based on the assumption of photons as real particles of non-zero and finite size, independently of usual electrodynamics, and certain historically postulated functional relationships between them were proved, that is, these functions can be derived mathematically strictly and consistently within the framework of the theory of the phenomenological radiative transfer if one takes the theory seriously by really assuming photons as particles. In this sense these functions may be treated as fundamental physical quantities within the scope of this theory, if one considers the possibility of the existence of photons.

1. Introduction

Maxwell's equations and Lorentz's force are the basis for one of the most successful physical theories: the Maxwell–Lorentz theory of electromagnetism. It considers dynamical vector fields and particles that are assumed to carry charges described by a charge density function. The medium between the charged particles is represented by two material functions. The mathematical relationships between the fields, the charge density and the material functions are given by Maxwell's equations and Lorentz's force equation. The fields are "generated by [charged] particles and they act on [charged] particles" [6, p. 26]. Maxwell's equations "present a set of pure phenomenological equations to describe the behavior of the electromagnetic fields in a certain region [...] [and] are 'just' a description of the action (the fields) but not of the causes (the sources)" [35, p. 198–199]. Furthermore, it should be mentioned the fact "that the Maxwell–Lorentz theory is the macroscopic description of the fundamental theory of electromagnetism" [6, p. 34]. The analytical properties as continuity and differentiability of the involved vector fields and functions do not follow from the theory, they must be assumed to satisfy the basic equations [35, p. 8].

The fields can be interpreted to describe radiation in the sense of an electromagnetic wave. Its absorption and scattering by the medium can then be quantified by solving Maxwell's equations within the framework of electromagnetic wave scattering [e.g. 4,23,5,24,35] and considering the scattering and absorbing medium (e.g. scattering objects) by spatially inhomogeneous material functions. These "material properties of the regions [...] must be therefore given, and cannot be derived within Electrodynamics itself" [35, p. 199]. But that is the typical situation in other physical theories as well. Finally, the transport of radiation can be calculated relatively exactly.

An independent and physically completely different approach to characterise the transport of radiation is given

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by the theory of radiative transfer. The basic idea is (i) to assume photons in order to exist as particles carrying energy, (ii) to define so-called state and process density functions in order to map the photon distribution and movement as well as the scattering and absorption properties of the traversed medium, respectively, and (iii) to establish an equation, the radiative transfer equation, whose derivation is usually performed with the help of a photon number balance consideration. This concept has a long history but is still present in the modern literature [e.g. 37,3,33,32,20,18,19,43,39,21,44,16,36,14,40]. The state density functions, as the so-called photon number density (PND), photon energy density (PED) and radiance, and the process density functions are introduced a priori as 'macroscopic' functions which contain the information of the photon movement as well as the underlying microscopic processes and may change with space, time and radiation direction due to sources and sinks of photons. But these processes on microscopic scale are not directly incorporated into the theory as also not the photons. In this sense the theory of radiative transfer is a macroscopic [33] and phenomenological [42] one and hence implies that 'radiation' is phenomenologically understood as the 'macroscopic' result of the directional movement of the 'microscopic' photons and the 'radiation field' is viewed as the spatio-temporal state of radiation characterised by the PND, PED and radiance. Thus, this theory is also a phenomenological and macroscopic one as the Maxwell–Lorentz theory, and the analytical properties of the density functions as boundedness, continuity, differentiability and integrability must also be tacitly assumed to establish radiative transfer equations. Moreover, it is also supposed that certain functional relations hold among the state density functions, for which, however, no strict derivations were presented within the scope of the theory itself.

Due to its insufficiencies it might seem that the theory of radiative transfer is not (i) fundamental because of too many a priori assumptions, (ii) complete because it is dealt with only model photons as particles which, however, do not appear directly in the theory and (iii) physically conceivable and reasonable because photons would not exist as particles – questions which are also of purely philosophical nature, of course. Moreover, these aspects are in close connection to recent developments in the theoretical research in the field of Maxwell–Lorentz theory. It has been shown that the latter theory is not only there to fill the process density functions, as the extinction coefficient and scattering phase function, of the radiative transfer equation, but to explain radiometric quantities as the radiance [e.g. 42,1] or to demonstrate the connection between both theories that vice versa the mathematical structure of the (scalar) radiative transfer equation may be derived from Maxwell–Lorentz theory, but also in a generalised vector formulation [e.g. 9,22,25,26,34]. For instance, in [25,26] it is stated that "the actual existence of the specific intensity as a fundamental physical quantity is postulated" and the radiance (specific intensity) "has no independent physical meaning" as well as "is not a fundamental physical quantity" and merely of "purely mathematical and auxiliary nature". The goal of the present paper is to reflect such statements: the existence of the radiance as originally postulated in the theory of radiative transfer has not to be assumed and its radiometric as well as analytical properties have not to be defined a priori as argued by the author of [26] but can be derived consistently *within* the scope of this theory, namely by taking the existence of photons as real particles seriously. If it works to consider them (along few additional but reasonable assumptions) successfully to derive a PND, PED and the radiance as originally postulated in the theory of radiative transfer, then this theory might still be viewed as fundamental, complete as well as physically conceivable and reasonable. Perhaps, one can also learn much more from this particle concept than it currently seems and will be discussed in the following? At least it should be allowed to discuss it scientifically.

With regard to the consideration of fields it should be mentioned that "Maxwell–Lorentz theory of electromagnetism works well (in the sense of describing physical phenomena correctly) when the fields are generated by smeared out charges (charge clouds), so one can describe the radiation from an antenna. It also works when the fields are given as 'external' fields, which act on charges by the Lorentz force equation […]. In short, electromagnetism is fine for most non-academic life. […]. But [the fundamental theory of electromagnetism] does not contain fields on the fundamental level" [6, p. 34]. The subsequent very worth reading Section 2.5 in that textbook discusses the Wheeler–Feynman theory of electromagnetism and why the Maxwell–Lorentz theory with its consideration of fields is mathematically inconsistent [6, p. 37]: "When we wish to explain a physical phenomenon, we reduce it (in the ideal case) to the behavior of the ontological quantities the physical theory is about. In Maxwell–Lorentz electromagnetism, fields are ontological. Switch on your radio. What better explanation is there than to say that the fields are out there, and they get absorbed as radio waves by the radio antenna, and that the radio transforms them back into air waves? Music to your ears. But in Wheeler–Feynman electromagnetism, there are no fields and only particles. It explains the music as well. But the explanation is different […]. If the Maxwell–Lorentz theory (with point charges) were mathematically consistent, we could cho[o]se between fields and particles as being 'real', or only particles as being 'real'. Since both would describe the macroscopic world as we see it, our choice would then have to be made on the grounds of simplicity and beauty of the theories. Perhaps in the future we shall find a simpler and nicer theory than the ones we have now, one which is solely about fields. Then only fields will be 'real'."

Beyond the electromagnetism, the consideration of real microscopic particles does not have to be in contrast to quantum theory as Bohmian mechanics demonstrates [6,7]. The same should also hold for photons. Of course, it is a "strange thing […] that neither Einstein […] nor ourselves know today what 'photons' really are. Are they particles? Are they extended objects? Are they anything at all?" [6, p. 123–124]. However, in the sense of "Whenever you say particle, mean it!" [6, p. v], the present paper just tries to take the particle picture seriously and assumes photons as real particles.

Summarising the preceding arguments the following scientific questions raise with regard to the subject to be investigated in the present paper: Is it possible to derive