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Application of the multigrid method in a deterministic solution scheme for the three-dimensional radiative transfer equation

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ABSTRACT

An improved solution scheme is developed for the three-dimensional radiative transfer equation (RTE) in inhomogeneous cloudy atmospheres. This solution scheme is deterministic (explicit) and utilizes spherical harmonics series expansion and the finite-volume method for discretization of the RTE. The first-order upwind finite difference is modified to take into account bidirectional flow of radiance in spherical harmonics space, and an iterative solution method is applied. The multigrid method, which is generally employed to achieve rapid convergence in iterative calculation, is incorporated into the solution scheme. The present study suggests that the restriction and prolongation procedure for the multigrid method must be also modified to account for bidirectional flow, and proposes an efficient bidirectional restriction/prolongation procedure that does not increase the computational effort for coarser grids, resulting in a type of wavelet low-pass filter. Several calculation examples for various atmosphere models indicate that the proposed solution scheme is effective for rapid convergence and suitable for obtaining adequate radiation fields in inhomogeneous cloudy atmospheres, although a comparison with the Monte Carlo method suggests that the radiances obtained by this solution scheme at certain angles tends to be smoother.

1. Introduction

The accurate calculation of radiative transfer in the atmosphere is an important issue in meteorological research. Realistic simulations of atmospheric and oceanic dynamics should take into account thermal energy transport, which is partly determined by radiative heating and cooling [1]. Remote sensing based on radiation measurements also requires radiative transfer calculations to be performed in order to carry out signal modeling, sensitivity tests, error estimation, and algorithm construction by estimating the relations between the optical properties of targets and the intensity of reflected and/or emitted radiance. However, there are certain difficulties associated with applying radiative transfer calculations to the atmosphere. One complication is related to the existence of clouds, which often introduces spatial inhomogeneity into the radiation field, due to the high spatial fluctuation of cloud particle density, changes of particle phase, and asymmetric scattering. Spatial inhomogeneity increases the complexity of the radiative transfer process, referred to as three-dimensional (3-D) effects, and results in lengthy computations. It also introduces large uncertainty in data retrieved through remote sensing (e.g., [2–4]).

Nevertheless, meteorological research cannot ignore radiative transfer in cloudy atmospheres, because the interaction between clouds and radiation is closely related

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to meteorological phenomena. A cloud layer modulates radiative heating and cooling of the atmosphere depending on its optical properties [5], which varies with changes in radiation that contributes the growth or decay of clouds [6]. Clouds are a significant factor in climate systems since they influence the radiation budget (e.g., [7]). Therefore, clouds should be taken into account, and continuous global observations should be performed, which also requires remote sensing techniques to be improved through quantitative examinations with radiative transfer calculations. It is, therefore, necessary to develop radiative transfer calculation schemes that are suitable for application to cloudy atmospheres with various types of clouds.

In general, radiative transfer calculations for a 3-D atmosphere are based on solving the 3-D radiative transfer equation (RTE), which is an expression of radiant energy conservation. Because solving the 3-D RTE for an arbitrary atmosphere usually requires considerable computational effort, many types of solution schemes have been examined in order to devise efficient and useful tools for radiation estimation in 3-D atmosphere models with the desirable accuracy. The fundamental and comprehensive knowledge necessary for finding a solution of the 3-D RTE, as well as its application to meteorological research, is presented in [8], and the history of the development of solution schemes is briefly summarized in [9]. Cahalan et al. [10] performed comparisons between many 3-D RTE solvers to verify the calculation results obtained from each solution scheme. Solution schemes for the 3-D RTE are broadly categorized into two groups: deterministic (or explicit) and stochastic (or probabilistic). Stochastic methods are used by most researchers investigating the 3-D RTE. Stochastic methods usually generate many trajectories of individual idealized energy packets, often regarded as "photons" for convenience (c.f. [11]), and statistically estimate radiative quantities such as reflection or absorption by counting propagated energy packets. The Monte Carlo method is usually used for trajectory calculations (e.g., [12]). A few deterministic methods have also been developed (e.g., [13–16]). In the deterministic approach, discretization of the 3-D RTE is usually carried out in order to separate variables. A discretized RTE can be solved by applying a general solution method for finite differential equations, such as a method used in fluid dynamics.

The suitability of each method depends on the conditions and purposes of the radiative transfer calculation, because deterministic and stochastic approaches have their individual merits and demerits. One of the merits of the deterministic approach is that it is generally capable of providing the entire spatial and angular distribution of the radiation field at once. This means that all radiative quantities, such as the intensity of radiance in an arbitrary direction, the net flux or the flux in a particular direction, can be derived from the same basic calculation result. Another merit is that it is free of statistical error, and therefore deterministic methods are highly suitable for sensitivity tests, which are used for quantitative estimation of the effects of changes in optical parameters on the radiation distribution. However, deterministic methods often require considerable computational resources and longer calculation times than stochastic methods to obtain equivalent results. It is then necessary to develop a suitable deterministic method that satisfies the conditions of both accuracy and feasibility required in various applications.

We have also developed a deterministic solution scheme [17] that utilizes an expansion in spherical harmonics for discretizing the radiation field in angle to reduce the computation time for the scattering integral term in the 3-D RTE. This technique is known as the P_n approximation. Ishida and Asano [17] also used the finite volume method to discretize the partial differential terms. Although the finite volume method is widely employed for solving various types of partial differential equations, this study revealed that a specific modification, referred to as the bidirectional upwind finite difference, must be applied to the 3-D RTE expanded by spherical harmonics in order to ensure that the equation is determinate. This method can provide reasonable results for radiative quantities, but there is still room for improvement, especially with respect to reducing the computation time.

Several iterative solution schemes for general finite difference equations employ the multigrid method, which can efficiently accelerate convergence. In brief, the multigrid method carries out iterative relaxation by projecting the equation onto a coarser grid than the original grid. This approach makes use of the idea that solution errors with low spatial frequency, which decay more slowly than high-frequency errors during the iteration of relaxation, can be converted into high-frequency errors as the equation is projected onto progressively coarser grids, expecting a rapid decay of the errors. Some solution schemes for radiative transfer also incorporate the multigrid method. For example, [18] developed a deterministic scheme discretized by the discrete ordinate method combined with the multigrid method, and suggested that the mathematical properties of the multigrid model match closely the physical properties of the radiation field. Thus it would be reasonable to apply the multigrid method to the solution scheme proposed by [17]. However, the general multigrid method is not suitable for the 3-D RTE expanded by spherical harmonics, and it must be modified in light of the reasons outlined in Section 2. Therefore, this article introduces a modification of the multigrid method and proves that this is efficient for solving the expanded 3-D RTE, especially for inhomogeneous cloudy atmospheres. In Section 2, we briefly explain the solution scheme proposed by [17] and outline the difficulties of application of the multigrid method to the 3-D RTE, as well as ways to overcome these difficulties. Section 3 shows some examples of calculation obtained by our solution scheme to verify its adequacy. The efficiency of the multigrid method and fundamentals of our solution scheme are discussed in Section 4, and a summary followed by conclusions is presented in Section 5.

2. Algorithm description

2.1. Solution scheme of Ishida and Asano (2007)

In this subsection, we briefly explain the solution scheme of [17], which is the starting point for developing