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A direct collocation meshless approach with upwind scheme for radiative transfer in strongly inhomogeneous media

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A B S T R A C T

A direct collocation meshless (DCM) method with upwind scheme is employed for solving the radiative transfer equation (RTE) for strongly inhomogeneous media. The trial function is constructed by a moving least-squares (MLS) approximation. The upwind scheme is implemented by moving the support domain of MLS approximation to the opposite direction of each streamline. To test computational accuracy and efficiency of the upwind direct collocation meshless (named UPCM) method, various problems in 1-D and 2-D geometries are analyzed. For the comparison, we also present cases of both the DCM method for the first-order RTE (employed by Tan et al. [1]) and the DCM for the MSORTE (a new second-order form of radiative transfer equation proposed by Zhao et al. [2]). The results show that the proposed method is more accurate and stable than the DCM method (no upwinding) based on both the RTE and MSORTE. Computationally, it is also faster.

1. Introduction

The radiative transfer equation (RTE) is an integro-differential equation widely used in different research areas to model wave motions such as light propagation through a turbulent atmosphere, electromagnetic waves propagating in plasmas, light propagation in biological tissue, or radiative heat transfer. Although it is a very well-known equation, analytical solutions are only available for the simplest problems. Therefore, numerical solutions to the RTE are necessary in practical applications.

Many numerical methods have been developed to solve the RTE in semitransparent media in recent years. These methods can be mainly classified into two kinds. The first kind of method is based on ray tracing, such as the ray tracing method [3,4], the zonal method [5,6], the DRESOR (Distributions of Ratios of Energy Scattered or Reflected) method [7], the Monte Carlo method [8–10], and the discrete transfer method [11–13]. These kinds of methods do not explicitly rely on the differential form of RTE because the simulation process of these methods is more physically based and the ray path is analytically determined. However, these methods have some obvious drawbacks, such as often being time-consuming even for relatively simple problems, or being difficult to deal with anisotropic scattering, or being difficult to be implemented to complex geometries due to the geometrical shielding. The second kind of method is based on the discretization of partial differential equations, such as spherical harmonics approximations [14,15], discrete ordinate methods (DOM) [16,17], the finite volume method (FVM) [18,19], the finite element method (FEM) [20,21], the collocation spectral method [22–24], and the Meshless method [25–27]. The methods based on the discretization of partial differential equation have the advantages of high efficiency and excellent flexibility to deal with multidimensional complex geometries. However, these methods suffer much from numerical oscillations in some cases caused by the convection-dominated property of the RTE.

The RTE is a first-order partial differential equation in each angular direction. Compared to the convection-diffusion

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Nomenclature

a	coefficients for MLS approximation
$\mathbf{a}$	vector of coefficient a
d	diameter of the support domain, m
I	radiation intensity, W/m^2 sr
i, j, k	general spatial indices
G	incident radiation, W/m^2
M	number of discrete directions
$\boldsymbol{n}$	unit normal vector
N_{sol}	total number of solution nodes
N	nodal basis function
$\mathbf{P}$	vector of legendre polynomial
p_j	legendre polynomial of jth order
q	heat flux, W/m^2
r	distance between points $\mathbf{x}$ and $\mathbf{x_i}$, m
S	source term of the radiative transfer equation
$\mathbf{s}$	unit vector in a given direction
t	computation time, s
T	temperature, K
V	solution domain
w	weight function
$\mathbf{x}$	vector of optical location

Greek symbols

β	extinction coefficient, m^{-1}
γ	upwind factor
κ_a	absorption coefficient, m^{-1}
κ_s	scattering coefficient, m^{-1}
ε	wall emissivity
μ_m, η_m, ξ_m	direction cosine in direction, m
λ	support domain amplifying factor
σ	Stefan-Boltzmann constant, W/m^2 K^4
τ	optical length
Φ	scattering phase function
$\mathbf{\Omega}$	vector of radiation direction
Ω	solid angle, sr
ω	scattering albedo

Subscripts

b	black body
i	node index
m	direction index
j, k	legendre polynomial order index
w	wall

Superscript

m, m'	direction index
T	transposition

equation, the RTE can be considered as a special kind of convection-dominated equation without the diffusion term. As mentioned before, the methods based on the discretization of partial differential equations may suffer from nonphysical oscillations in the numerical results, especially for the models with inhomogeneous media where some regions have very small/zero extinction coefficient or discontinuous extinction coefficient. To make the numerical discretization schemes correctly model the transfer process, two kinds of special stabilization techniques are often used: (1) transforming the RTE into a numerical stable equation, for example, the second-order partial differential equation, and (2) taking various numerical stabilization schemes, such as upwind scheme or artificial diffusion which are often used in FDM (finite difference method), FVM, FEM and the meshless method.

The second-order form of radiative transfer equation (SORTE) contains second-order derivative terms which are known to have the characteristic of diffusion and good numerical properties. Besides, the diffusion term introduced in the analytical transformation process is natural and consistent with the original first-order equation. In recent years, three different transformed equations have been proposed [2,28–32]. One is the even-parity (EPRTE) formulation of the RTE mentioned in Refs. [28–30] which is a second-order partial differential equation of the even parity of radiative intensity. Another is the second-order radiative transfer equation (SORTE) proposed recently [31,32] which uses radiative intensity as solution variable and is more convenient to be applied to the complex radiative transfer equation, but has some problems in dealing with inhomogeneous media due to the existence of the reciprocal of extinction coefficient in the equation. The third is a new second-order form of radiative transfer equation (named MSORTE) proposed to overcome the singularity problem of the SORTE. Zhao et al. [2] applied a meshless method based on the moving least square approximation to verify the versatility and performance of the MSORTE in solving radiative transfer in very strongly inhomogeneous media, and stable and accurate results were obtained. Although the three types of SORTE are more stable than RTE, the iterative solutions of the SORTE are more time consuming. In addition, compared to the first-order RTE, SORTE is of higher differentiability of radiative intensity with respect to the ray trajectory coordinates, which will limit the application of SORTE.

An alternative approach to reduce the nonphysical oscillations in the results is the introduction of RTE with the upwind scheme which can obtain the numerical stable solutions under less computational cost compared to SORTE. A number of upwinding schemes have been developed for FDM, FEM and FVM. It is apparent that the upwind effect, achieved by whatever means, is needed only in the direction of flow. However, it is not easy to design such methods for multidimensional cases. Hughes and Brooks [33] introduced the ‘Streamline Upwind (SU) method’ where the artificial diffusion operator was constructed to act only in the flow direction. However, the SU method cannot meet the consistency of the RTE, resulting in excessively diffuse solutions, and at the cost of accuracy. Hughes and Brooks [34] proposed an improved upwind