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Polarization-induced coherence changes and conditions for the invariance of the spectral degree of coherence produced by an electromagnetic wave scattering on a collection of particles

Jingjing Chen, Feinan Chen, Yanru Chen, Yu Xin, Yanming Shao, Qi Zhao *

Department of Optical Engineering, Nanjing University of Science and Technology, Nanjing 210094, Jiangsu Province, China

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ABSTRACT

The scattered field generated by an arbitrary polarized electromagnetic plane wave incident on a collection of particles is studied. First, formulas for the spectral density and spectral degree of coherence of the scattered field are derived. Then, the polarization-induced coherence changes of a spatially coherent electromagnetic light wave scattered from a collection of particles are explored, and the results show that the spectral coherence of the scattered field consists of three independent parts. One depends on the polarization properties of the incident wave; another depends on the scattering potential of the particle; the third depends on the generalized pair-structure factor of the particle system. Lastly, the conditions for the invariant spectral degree of coherence of an arbitrary polarized wave scattered from a collection of particles are discussed. Apart from the geometrical factor, the modulus of the generalized pair-structure factor of the particle system and the modulus of the correlation function of the scattering potential can be factorized.

1. Introduction

The scattering of electromagnetic waves from collections of particles is of considerable interest in many areas such as medical diagnostics, remote sensing and so on. The scattering of a light wave from a collection of particles was studied in [1] and the results suggest that spectral changes produced by scattering could be used to determine the structure of some scattering systems. The scattering theory has been generalized to the stochastic scalar fields [2]. Later a novel model dealing with the scattered light of scalar fields from collections of particles was set up [3]. After that, the effect of the pair-structure factor of the particulate medium on the scattering properties was analyzed [4], which was based on the scalar theory and imposed serious limitations on the applicability of the results. Meanwhile, the inverse problem, i.e., the determination of the density correlation functions for a collection of particles was studied [5], and the method to determine the pair-structure factor of a collection of particles was discussed [6]. Besides, the cross-spectral density function of the scattered field produced by a coherent plane light wave incident on a collection of different types of anisotropic particles was derived in [7,8]. In the past two decades, numerous papers were published on the analysis of scattering by deterministic or random distributions of identical or disparate particles. Moreover, the spectral coherence of the scattered field, which may provide some useful information about the scattering medium, is very important in the inverse problem. Therefore, it is necessary to know the full properties of the scattered field, especially the degree of spectral coherence. Correlation-induced and polarization-induced coherence changes of an electromagnetic wave on scattering were outlined

* Corresponding author. Tel.: +86 2584315435.
E-mail address: zhaoqi@njust.edu.cn (Q. Zhao).

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in [9,10]. Furthermore, the conditions for the invariance of the spectral degree of coherence of a coherent scalar plane wave while it is scattered from random media were obtained in [11], and then the conditions for the scalar wave were extended to a more general three-dimensional electromagnetic wave case in [12]. On the other hand, the beamlike condition for a plane wave was obtained while it is scattered on a stochastic medium [13]. But the polarization-induced coherence changes produced by scattering from a static system of particles, to the best of our knowledge, have not been reported. In this paper, within the accuracy of the first-order Born approximation, the polarization-induced coherence changes of a spatially coherent electromagnetic wave scattered from a collection of particles are studied. Specifically, the conditions for the invariant spectral degree of coherence of an arbitrary polarized plane wave scattered from a collection of particles are obtained.

2. An electromagnetic wave incident on a collection of particles

Consider a spatially coherent electromagnetic plane wave incident on a collection of particles that occupy domain V in a direction specified by a real unit vector $\mathbf{s}_0$. Take the directional of the z-axis of the Cartesian coordinate along $\mathbf{s}_0$, and $\mathbf{s}$ is the unit vector of the wave vector. The other Cartesian coordinates are chosen as

$$\mathbf{x}=\frac{\mathbf{s}\times\mathbf{s}_0}{|\mathbf{s}\times\mathbf{s}_0|},\quad \mathbf{y}=\mathbf{z}\times\mathbf{x}=\mathbf{s}_0\times\frac{\mathbf{s}\times\mathbf{s}_0}{|\mathbf{s}\times\mathbf{s}_0|} \tag{1}$$

Thus, the incident wave can be expressed as [14,15]

$$\mathbf{E}^{(i)}(\mathbf{r},t)=\mathbf{E}^{(i)}(\mathbf{r},\omega)\times\exp(-i\omega t), \tag{2}$$

$$\mathbf{E}^{(i)}(\mathbf{r},\omega)=[E_x^{(i)}(\mathbf{r},\omega),E_y^{(i)}(\mathbf{r},\omega)], \tag{3}$$

where

$$E_x^{(i)}(\mathbf{r},\omega)=a(\omega)\times\exp(ik\mathbf{s}_0\cdot\mathbf{r}),E_y^{(i)}(\mathbf{r},\omega)=b(\omega)\times\exp(ik\mathbf{s}_0\cdot\mathbf{r}) \tag{4}$$

Here, $k=2\pi/\lambda$ is the wave number, $\mathbf{r}$ is a position vector of a field point, superscript (i) denotes the incident field.

In order to characterize the response of such a collection to the incident light, we will use the discrete-particle model in which the scattering potential $F(\mathbf{r}',\omega)$ of the collection can be represented by a finite sum of potentials of the individual scatters. Assume that there are L different types of particles forming the system, M_l of each type $(l=1,2,\ldots L)$, deterministically located at points specified by position vectors $\mathbf{r}_{lm}$. f_l is the scattering potential of the scatterer of type l. In the case when the collection is static but random one can characterize its response with the help of the correlation function of the scattering potential which reduces to the form [2]

$$C^{(F)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)=\langle F^*(\mathbf{r}'_1,\omega)F(\mathbf{r}'_2,\omega)\rangle_{rm}=\sum_{l=1}^{L}\sum_{j=1}^{L}\sum_{m=1}^{M(l)}\sum_{n=1}^{N(j)}\langle f_l^*(\mathbf{r}'_1-\mathbf{r}_{lm},\omega)f_j(\mathbf{r}'_2-\mathbf{r}_{jn},\omega)\rangle_{rm} \tag{5}$$

where $\langle\rangle_{rm}$ denotes the average taken over the ensemble of realizations of the scattering medium.

For the sake of simplicity, we assume that the particles are identical, Eq. (5) then becomes

$$C^{(F)}(\mathbf{r}'_1,\mathbf{r}'_2,\omega)=\sum_{m=1}^{M}\sum_{n=1}^{N}\langle f^*(\mathbf{r}'_1-\mathbf{r}_m,\omega)f(\mathbf{r}'_2-\mathbf{r}_n,\omega)\rangle_{rm} \tag{6}$$

In particular, the scattering can be characterized by an ordinary scattering matrix which describes the change in the amplitude of a plane wave incident along $\mathbf{s}_0$ and scattered along $\mathbf{s}$. Within the accuracy of the first-order Born approximation, the scattering matrix is given by the simple expression

$$\mathbb{S}(\mathbf{s}_0,\mathbf{s};\omega)=\widetilde{F}[k(\mathbf{s}-\mathbf{s}_0);\omega] \tag{7}$$

For a description of scattering from a random medium, the spectral pair-scattering matrix should be employed [3]

$$\mathbb{M}(\mathbf{K}_1,\mathbf{K}_2,\omega)=\langle\mathbb{S}^*(\mathbf{K}_1,\omega)\mathbb{S}(\mathbf{K}_2,\omega)\rangle_{rm}=\widetilde{C^{(F)}}(\mathbf{K}_1,\mathbf{K}_2,\omega) \tag{8}$$

where $\mathbf{K}_1=k(\mathbf{s}_1-\mathbf{s}_0)$, $\mathbf{K}_2=k(\mathbf{s}_2-\mathbf{s}_0)$.

Substituting Eq. (6) into Eq. (8) and evaluating the Fourier transforms of individual particles in the collection with the help of the variables $\mathbf{R}_{2n}=\mathbf{r}_2-\mathbf{r}_n$ and $\mathbf{R}_{1m}=\mathbf{r}_1-\mathbf{r}_m$, we find that

$$\mathbb{M}(\mathbf{K}_1,\mathbf{K}_2,\omega)=\widetilde{C_f}(\mathbf{K}_1,\mathbf{K}_2,\omega)\mathbb{Q}(\mathbf{K}_1,\mathbf{K}_2) \tag{9}$$

where

$$\widetilde{C_f}(\mathbf{K}_1,\mathbf{K}_2,\omega)=\iint\langle f^*(\mathbf{R}_{1m},\omega)f(\mathbf{R}_{2n},\omega)\rangle_{rm}e^{-i(\mathbf{R}_{2n}\cdot\mathbf{K}_2-\mathbf{R}_{1m}\cdot\mathbf{K}_1)}d^3\mathbf{R}_{1m}d^3\mathbf{R}_{2n} \tag{10}$$

and

$$\mathbb{Q}(\mathbf{K}_1,\mathbf{K}_2,\omega)=\left\langle\sum_{n=1}^{N}\sum_{m=1}^{N}e^{-i(\mathbf{r}_m\cdot\mathbf{K}_2-\mathbf{r}_n\cdot\mathbf{K}_1)}\right\rangle_{rm} \tag{11}$$

is a generalized pair-structure factor of the particle system. This provides the measure of the correlation (similarity) between waves along transfer vectors $\mathbf{K}_1$ and $\mathbf{K}_2$. The pair-structure factor reduces to the ordinary structure factor $\mathcal{S}(\mathbf{K},\omega)$ if the momentum transfer vectors coincide

$$\mathcal{S}(\mathbf{K},\omega)=\left\langle\sum_{n=1}^{N}\sum_{m=1}^{N}e^{-i\mathbf{K}(\mathbf{r}_m-\mathbf{r}_n)}\right\rangle_{rm} \tag{12}$$

Another complex-valued quantity, which may be regarded as a degree of angular correlation should be introduced [4]

$$q(\mathbf{K}_1,\mathbf{K}_2,\omega)=\frac{\mathbb{Q}(\mathbf{K}_1,\mathbf{K}_2;\omega)}{\sqrt{\mathcal{S}(\mathbf{K}_1;\omega)}\sqrt{\mathcal{S}(\mathbf{K}_2;\omega)}} \tag{13}$$

This measure is similar to the spectral degree of coherence of optical fields but describes the ability of a random medium to "decorrelate" waves and its absolute value varies between 0 and 1.

Within the accuracy of the first-order Born approximation [8,16], the cross-spectral density matrix of the scattered field is derived

$$\mathbf{W}^{(s)}(\mathbf{r}_1,\mathbf{r}_2,\omega)=\frac{\mathbf{d}\cdot\exp[ik(r'_2-r'_1)]}{r_1r_2}\int_D\int_D\langle F^*(\mathbf{r}'_1,\omega)F(\mathbf{r}'_2,\omega)\rangle\times\exp[ik\mathbf{s}_0\cdot(\mathbf{r}'_2-\mathbf{r}'_1)]\times\exp[ik(\mathbf{s}_1\cdot\mathbf{r}'_1-\mathbf{s}_2\cdot\mathbf{r}'_2)]\mathrm{d}^3r'_1\mathrm{d}^3r'_2 \tag{14}$$