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# Polarimetric and angular light-scattering from dense media: Comparison of a vectorial radiative transfer model with analytical, stochastic and experimental approaches

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ABSTRACT

Our work presents computations via a vectorial radiative transfer model of the polarimetric and angular light scattered by a stratified dense medium with small and intermediate optical thickness. We report the validation of this model using analytical results and different computational methods like stochastic algorithms. Moreover, we check the model with experimental data from a specific scatterometer developed at the Onera. The advantages and disadvantages of a radiative approach are discussed. This paper represents a step toward the characterization of particles in dense media involving multiple scattering.



## 1. Introduction

The study of polarimetric and angular light-scattering from single particles is of interest in scientific fields such as remote-sensing, particle characterization, and atmospheric models [1–4]. Modeling and measuring the scattered intensities are relevant in fields ranging from material science to remote-sensing [5–7]. The problem of light-scattering from dense media is more complex as it involves multiple scattering. Different numerical approaches, such as vectorial radiative transfer or stochastic methods, were developed to compute the global signature of dense scattering media.

The polarimetric scattered-light intensity is described in the Stokes vector and Mueller matrix framework [8,9], and enhances direct relationships between theory and measurements. The Stokes vector $\vec{S} = (I, Q, U, V)^T$ is written as a sum of horizontal and vertical electric field products [10]. The scattering media of interest consist of an ensemble of particles embedded in a non-scattering host media. The interaction of light with a scattering medium transforms the incident Stokes vector $\vec{S}$ into an emerging vector $\vec{S}'$ as $\vec{S}' = \mathbf{P} \cdot \vec{S}$, where $\mathbf{P}$ is the phase matrix. This matrix can be computed from the single particle matrix modeled directly using the T-matrix method [11–13], and depends on the relative refractive index, size distribution, and morphology of the particle.

In this paper, we report analytical, numerical, and experimental validation of a radiative transfer model adapted to a dense plane-parallel scattering media in a slab. We first solve the vectorial radiative transfer equation (VRTE) using an *adding–doubling* method. Analytical validation is carried using Coulson's table. We also report a numerical validation using a stochastic method. Finally, the radiative transfer is compared to laboratory measurements on nanoparticles in water using a scatterometer developed at Onera, The French Aerospace Lab.

## 2. Vectorial radiative transfer model

The propagation of polarized light in dense scattering media can be computed by solving the VRTE [14]. Traditionally, this is considered a phenomenological equation

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based on classical radiometry and polarimetry; yet it was recently derived from the Maxwell equations using a microphysical approach [15]. It is worth noting, however, that some electromagnetic effects are not taken into account in the radiative transfer theory [14,15].

Let us consider the vectorial radiance $\vec{L}(\vec{x},\vec{s})$ in the Stokes formalism where $\vec{x}$ is the position and $\vec{s}$ is the orientation of the observation point. The static and monochromatic VRTE for 3D dense scattering media with randomly-oriented particles is usually written as

$$\vec{s}\cdot\nabla\vec{L}(\vec{x},\vec{s})+\mathbf{K}_{ext}(\vec{x})\cdot\vec{L}(\vec{x},\vec{s})=\mathbf{K}_{sca}(\vec{x})\cdot\int_{\Omega}\mathbf{P}(\vec{x},\vec{s},\vec{s}')\cdot\vec{L}(\vec{x},\vec{s}')\cdot d\varpi(\vec{s}')$$

where $\mathbf{K}_{ext}(\vec{x})$ and $\mathbf{K}_{sca}(\vec{x})$ are the extinction and scattering coefficient matrices. Also, $\mathbf{P}(\vec{x},\vec{s},\vec{s}')$ is the phase matrix and $d\varpi(\vec{s}')$ is the solid angle. We note that the thermal radiation term is negligible at ambient temperature for the visible spectrum.

We describe the plane-parallel vectorial radiative transfer scheme (Metropol) for a stratified scattering media. The general description of the atmospheric layers described by Evans and Stephans [16] for remote sensing applications is substituted here by interfaces with fixed refractive indexes (Fig. 1). Evans models a finite-thickness homogeneous glass slab [17–19]. We assume the medium inside the slab is composed of distinct scattering layers. We associate with each layer of this medium the radiative parameters: optical thickness $\tau$, single-scattering albedo $\omega$ and phase matrix $\mathbf{P}$ [20]. As we aim to model scattering media in cuvette samples, Fresnel equations must be solved at each interface (e.g. air/glass/water).

We now numerically solve the VRTE for a general system. A Fourier decomposition is used to represent azimuth angles, following [1,21,22]. We take into account multiple reflections between layers in the polarimetric radiance balance. The *adding–doubling* technique [23] is

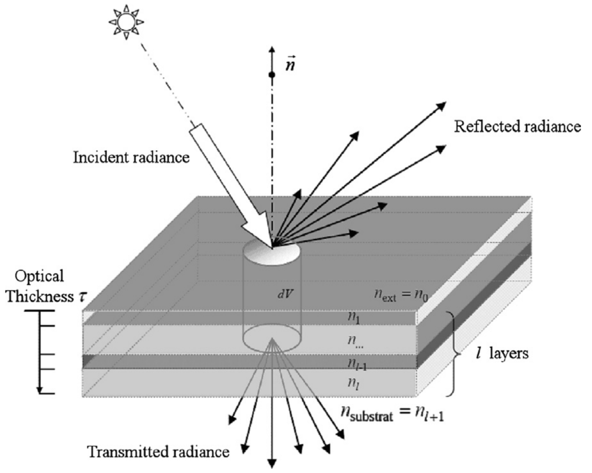


**Fig. 1.** Schematic of a generic dense scattering medium modeled as semi-infinite multiple layers.

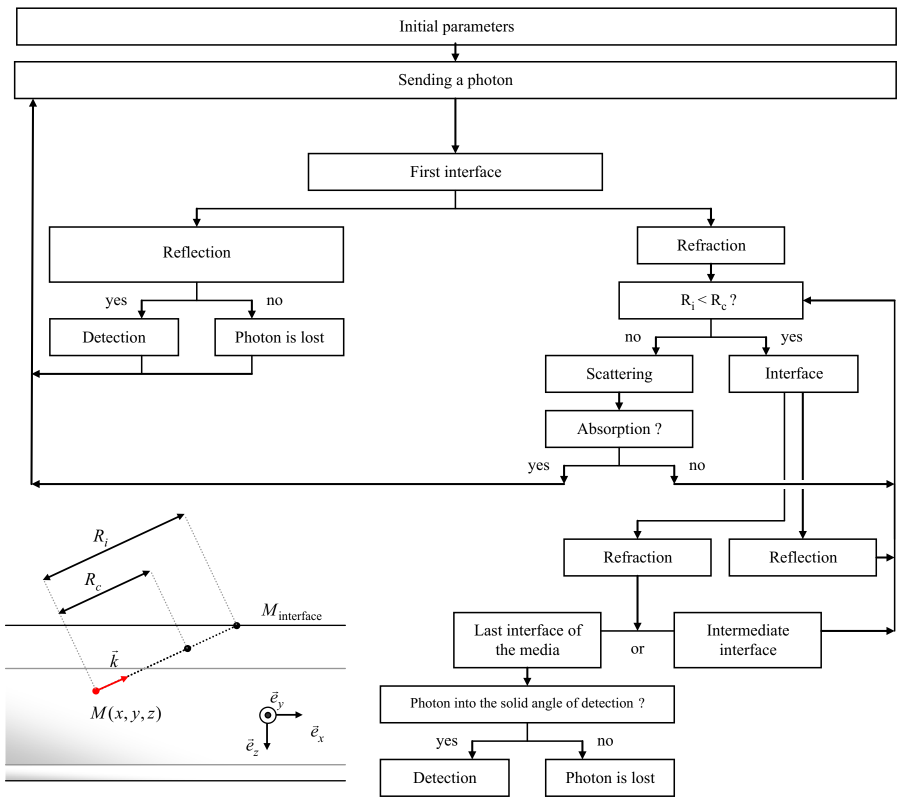


**Fig. 2.** General Monte Carlo scheme of a multi-layered media. Reflection and refraction are considered at each interface. Polarized scattering events include the possibility of light absorption by the scatter. The incident Stokes vector is modified by the $4\times4$ Mueller matrix.