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Modeling variability in dendritic ice crystal backscattering cross sections at millimeter wavelengths using a modified Rayleigh–Gans theory

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ABSTRACT

Using the Generalized Multi-particle Mie-method (GMM), Botta et al. (in this issue) [7] created a database of backscattering cross sections for 412 different ice crystal dendrites at X-, Ka- and W-band wavelengths for different incident angles. The Rayleigh–Gans theory, which accounts for interference effects but ignores interactions between different parts of an ice crystal, explains much, but not all, of the variability in the database of backscattering cross sections. Differences between it and the GMM range from −3.5 dB to +2.5 dB and are highly dependent on the incident angle. To explain the residual variability a physically intuitive iterative method was developed to estimate the internal electric field within an ice crystal that accounts for interactions between the neighboring regions within it. After modifying the Rayleigh–Gans theory using this estimated internal electric field, the difference between the estimated backscattering cross sections and those from the GMM method decreased to within 0.5 dB for most of the ice crystals. The largest percentage differences occur when the form factor from the Rayleigh–Gans theory is close to zero. Both interference effects and neighbor interactions are sensitive to the morphology of ice crystals. Improvements in ice-microphysical models are necessary to predict or diagnose internal structures within ice crystals to aid in more accurate interpretation of radar returns. Observations of the morphology of ice crystals are, in turn, necessary to guide the development of such ice-microphysical models and to better understand the statistical properties of ice crystal morphologies in different environmental conditions.

1. Introduction

Ice crystals in the atmosphere grow into various shapes, sizes and masses. These properties of ice crystals in turn determine their potential for future growth and fall speeds, hence interactions with other ice crystals and cloud drops, which have a bearing on the lifetimes of clouds (see Part IV of Lamb and Verlinde [1]). These properties of ice crystals also determine their visible and infrared properties which impact the energy budget of the atmosphere. Millimeter-wave radar signals from ice crystals, such as backscattered powers and differential reflectivity, are valuable in probing ice crystal properties within clouds optically thick at visible and infrared wavelengths. Cloud models can utilize a forward model to calculate ice crystal radiation properties and map model outputs of ice crystal properties into radar signals. For these applications, both the number fraction of each type of ice crystal within a model volume and the forward model used to calculate

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the radiation properties of ice crystals from model outputs of ice crystal shapes, sizes and masses are critical.

Most cloud models in use today track only two pieces of information regarding modeled ice crystals: mass and maximum dimension. Sulia and Harrington [2] argue that this is insufficient for accurate modeling of the evolution of different populations of ice crystals; they obtain more realistic model results by tracking mass and the two dimensions of an underlying spheroidal shape for ice crystals. Treating ice crystals as spheroids in computation of their backscattering cross sections at millimeter wavelengths has a history within the atmospheric radiation community (e.g. Matrosov [3]), however recent results of Botta et al. [4] question this approach. They modeled the complicated shapes of ice crystal aggregates with different shapes, sizes and masses as collections of thousands of tiny (about two orders of magnitude smaller than the wavelength), non-overlapping, closely packed spheres. They subsequently used the Generalized Multi-particle Mie method (GMM [5,6], a numerical method that computes the scattering properties of clusters of non-overlapping spheres) to calculate the backscattering cross sections of these aggregates. They learned that aggregates with similar masses and maximum dimensions can have backscattering cross sections that vary by tens of dB's. Modeling aggregates as spheroids with effective dielectric constants, which is often used in practice today, is incapable of capturing this variability and can lead to errors as large as tens of dB's.

Botta et al. [7] extended the Botta et al. [4] results to dendrites by creating 412 different realizations of a dendrite using from 2659 to 49,879 tiny spheres packed into three layers of a face-centered cubic (FCC) lattice. These dendrites have eleven different maximum dimensions from ~0.5 mm to ~5.5 mm, equally partitioned in logarithmic space. The thickness of the dendrites was based on the thickness-size relationship for P1e crystals from Pruppacher and Klett [8] together with variations of 1/2 of that thickness to increase the range of variability. Dendrites with the same maximum dimension and thickness have different widths, core sizes, branch widths, sub-branch numbers and locations. (See Fig. 1 for an example of a dendrite composed of 2659 tiny spheres. Fig. 1 in Botta et al. [7] contains examples of their constructed dendrites compared with real dendrites; their appendix contains detailed information on dendrite geometries.) The masses of the dendrites are determined by, hence overlap with, a range of representative mass-dimensional relationships (see Fig. 2 in Botta et al. [7]). Because the closely packed spheres occupy only 74% of the overall dendrite volume, the scattering properties of clusters of tiny spheres with dendrite-like shapes are calculated. Botta et al. [7] show that it is reasonable to increase the thickness of a GMM model dendrite to match the mass and maximum dimension of a real dendrite.

Each dendrite is illuminated by both horizontally and vertically polarized radiation at W-band (3.19 mm), Ka-band (8.40 mm) and X-band (31.86 mm) wavelengths. The dielectric constant of solid ice at 0 °C is used: 3.1682+i 3.2586×10^{-4} for W-band, 3.1683+i 6.5053×10^{-4} for Ka-band and 3.1688+i 1.6777×10^{-3} for X-band. The illumination

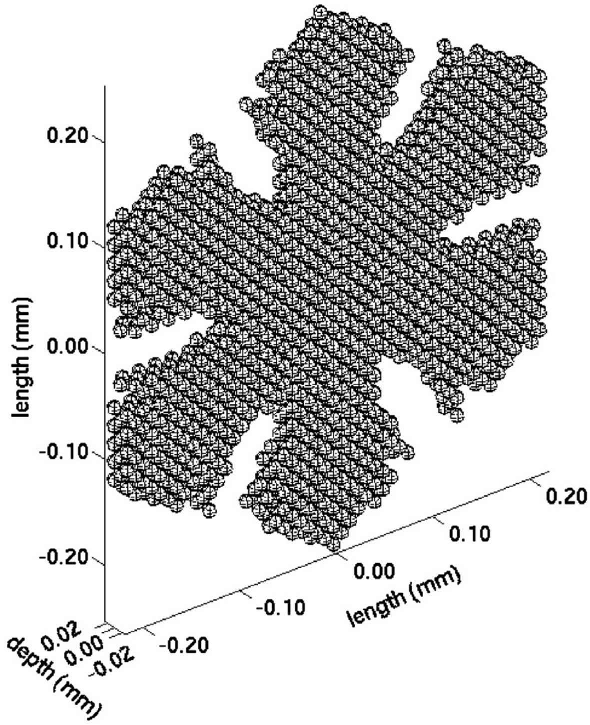

Fig. 1. Example of a dendrite in the database of Botta et al. [7]. This dendrite is made of three layers composed of 2659 tiny spheres arranged in a face-centered cubic lattice.

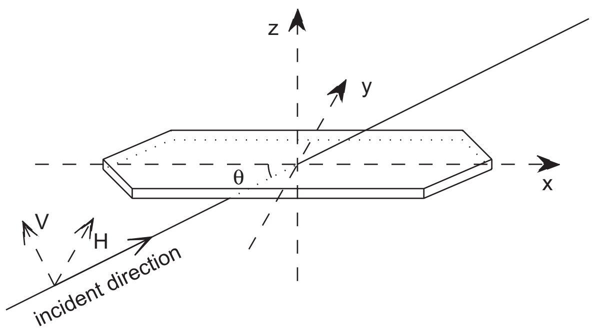

Fig. 2. Angle of incidence θ and polarization states H and V of radiation illuminating an ice crystal. Vertical incidence corresponds to a 90° incident angle while side incidence corresponds to a 0° incident angle.

ranges from perpendicular (side incidence) to parallel (vertical incidence) to the ice crystal symmetry axis with several angles in-between (Fig. 2). The large spread in backscattering cross sections that result from Botta et al.'s [7] GMM calculations is illustrated for side incidence, vertical incidence and two angles in-between in Fig. 3 for *hh* polarization (i.e. illuminating with h-polarization waves and measuring h-polarization returns) backscattering cross sections σ_{hh}. Fig. 4 contains the results for *vv* polarization backscattering cross sections σ_{vv}. These two figures show that for dendrites with the same maximum dimensions and incident directions of illumination the spread in the backscattering cross sections can be tens of dB's, especially for dendrites larger than half of the wavelength. These results of Botta et al. [7,4] suggest that while