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Variability in millimeter wave scattering properties of dendritic ice crystals

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ABSTRACT

A detailed electromagnetic scattering model for ice crystals is necessary for calculating radar reflectivity from cloud resolving model output in any radar simulator. The radar reflectivity depends on the backscattering cross sections and size distributions of particles in the radar resolution volume. The backscattering cross section depends on the size, mass and distribution of mass within the crystal. Most of the available electromagnetic scattering data for ice hydrometeors rely on simple ice crystal types and a single mass–dimensional relationship for a given type. However, a literature survey reveals that the mass–dimensional relationships for dendrites cover a relatively broad region in the mass–dimensional plane. This variability of mass and mass distribution of dendritic ice crystals cause significant variability in their backscattering cross sections, more than 10 dB for all sizes (0.5–5 mm maximum dimension) and exceeding 20 dB for the larger ones at X-, Ka-, and W-band frequencies. Realistic particle size distributions are used to calculate radar reflectivity and ice water content (*IWC*) for three mass–dimensional relationships. The uncertainty in the *IWC* for a given reflectivity spans an order of magnitude in value at all three frequencies because of variations in the unknown mass–dimensional relationship and particle size distribution. The sensitivity to the particle size distribution is reduced through the use of dual frequency reflectivity ratios, e.g., Ka- and W-band frequencies, together with the reflectivity at one of the frequencies for estimating *IWC*.

1. Introduction

Electromagnetic scattering models of hydrometeors are necessary to perform calculation of radar observables. Great care must be given to the fidelity of the hydrometeor models used in such computations in order to accurately reproduce particle characteristics such as their shape, size and mass. Ice hydrometeors such as pristine crystals are particularly challenging from a modeling perspective because their morphological characteristics exhibit large natural variability. Of this natural variability only limited aspects have been quantified in the literature: typically mass–dimensional relationships and thickness–size relationships [1] (p. 51). Even for a single class of pristine ice crystal, e.g., columns or dendrites, the mass–dimensional relationships available in the literature have large variability, as summarized by Avramov and Harrington [2]. Moreover, cursory inspection of the data used to obtain even a single mass–dimensional relationship reveal large scatter about the fitted expression.

Electromagnetic scattering computations for ice particles must capture the variability of all hydrometeor characteristics in order to provide accurate and useful results. This paper presents scattering computations for dendritic ice crystals. Dendritic crystals are particularly challenging because of the great variability of their natural shapes: they

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have variations in size of the hexagonal core, width and length of branches, and often articulate sub-branches, to mention just a few distinctive traits. This multitude of shape variations leads to a wide range of mass, shape and projected area for a crystal of given size. It will be shown that radar backscattering cross sections are sensitive to these variations and that the radar reflectivity is sensitive to the particular choice of mass–dimensional relationship and to the particle size distribution.

The scattering properties of pristine ice crystals for radar applications have been investigated by the scientific community. Various scattering computation techniques have been employed to model the complex morphological structure of such particles. The discrete dipole approximation (DDA; e.g., [3,4]), the finite difference time domain (FDTD; [5,6]) method and the generalized multi-particle Mie (GMM; [7]) method were all successfully applied to compute the scattering parameters of pristine ice crystals at microwave frequencies (e.g., [8], FDTD; [9,10], DDA; [11,12], GMM). The common denominator of all of these studies is the use of a single mass–dimensional relationship for each individual type of crystal. This approach is popular because it allows treating the backscattering cross section as a single-valued function of size, noticeably reducing the overall complexity of the scattering model and the time needed for the scattering computations. This approach is a carryover from the treatment of raindrops for which the equilibrium shape, and thus mass, is a well-defined function of size and variations around the equilibrium are negligible for the purpose of scattering computations.

In this work an accurate model of the microphysical morphology of dendritic ice crystals is coupled to the GMM scattering model in an effort to capture as much as possible the natural variability of dendritic crystal scattering properties. Single particle scattering computations at multiple radar wavelengths and multiple incidence angles are presented. Simulated particle size distributions based on the modified gamma distribution are used to explore the variability of the radar reflectivity and dual frequency ratios for different mass–dimensional relationships.

2. Electromagnetic scattering method and hydrometeor model

2.1. The GMM method for ice hydrometeor scattering computations

The GMM method is an analytical solution of Maxwell's equations in free space for a cluster of non-overlapping spheres having arbitrary size, composition and location [7]. This method proved to be in good agreement with measurements [13]. Botta et al. [12] have compared backscattering cross sections of oblate spheroids using GMM and T-matrix methods, which show good agreement (less than 1 dB). The GMM method was used for the computations of scattering parameters of ice crystals, aggregates and melting aggregates by Grecu and Olson [11] and Botta et al. [12,14]. The necessary assumption to use GMM is that the scattering target can be reasonably approximated by a cluster of spheres. This assumption is valid as long as the cluster of spheres captures the general size and shape of and distribution of mass within the particle, i.e., the mass in the core, branches and sub-branches.

A reasonable approach to modeling a solid particle as a cluster of spheres is by using a sphere packing strategy to match size and shape. Such strategies ensure maximum packing factor, i.e., the ratio between the volume occupied by the spheres and the total volume. The simplest close packing of spheres consists of creating a regular lattice of equal size spheres. There are two simple regular lattices that achieve maximum packing factor: the face centered cubic lattice (FCC) and the hexagonal close packing lattice (HCP). They can both be thought of as stacks of identical planar layers composed of spheres arranged at the vertices of a triangular tiling. The FCC and HCP lattices differ in how the layers are stacked on top of each other. In both lattices each sphere has 12 neighbors and they both achieve the maximum theoretical packing factor for equal sphere lattices, $\pi/\sqrt{18}\sim 0.74$. Such a strategy produces targets of the correct size and shape but with underestimated mass density (74% of the individual sphere density).

In order to obtain the correct value of mass density, the gaps in the lattice may be filled recursively by increasingly smaller spheres. However, with this approach the mass density increases logarithmically with the number of spheres added to the lattice, e.g., increasing the mass density to 90% of the correct value requires between 2 and 3 orders of magnitude increase in the number of spheres in the lattice. The computation time for the GMM code scales with the square of the total number of spheres in the cluster, therefore, improved model fidelity comes at increasing computational cost.

To keep this work feasible it is necessary to compromise between the exact particle shape and its mass density. Because the scattering property of ice crystals are in large part determined by their total mass and their maximum dimension, it was chosen to exactly match these two quantities at the cost of lower mass density. To test the effect of this compromise, GMM simulations for stellar crystals (P1d) composed of three layers of the FCC lattice were compared to results obtained by Aydin and Walsh [8] using the FDTD method. The GMM model ice crystals had the same mass, maximum dimension and branch width as those used by Aydin and Walsh [8] but were made slightly thicker. This approximation is justified because the particle thickness is much smaller than the wavelength and the particle's maximum dimension. The GMM backscattering cross sections at vertical incidence matched the FDTD ones to within about 2 dB for crystals with maximum dimension between 0.2 mm and 2 mm at both Ka- and W-band. Therefore the GMM backscattering cross sections are assumed to be representative of natural particles of equivalent mass and maximum dimension.

2.2. Pristine crystals model

The pristine crystals modeled in this work are of dendritic type. Crystals of maximum dimension of 11 logarithmically spaced values ranging from 0.5 mm to about 5.5 mm were defined. The crystal thickness is defined by