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Minimum principles in electromagnetic scattering by small aspherical particles

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ABSTRACT

We consider the question of optimal shapes, e.g., those causing minimal extinction among all shapes of equal volume. Guided by the isoperimetric property of a sphere, relevant in the geometrical optics limit of scattering by large particles, we examine an analogous question in the low frequency approximation, seeking to disentangle electric and geometric contributions. To that end, we survey the literature on shape functionals and focus on ellipsoids, giving a simple discussion of spherical optimality for the coated ellipsoidal particle. Monotonic increase with asphericity in the low frequency regime for orientation-averaged induced dipole moments and scattering cross-sections is also shown. Additional physical insight is obtained from the Rayleigh–Gans (transparent) limit and eccentricity expansions. We propose connecting low and high frequency regimes in a single minimum principle valid for all size parameters, provided that reasonable size distributions of randomly oriented aspherical particles wash out the resonances for intermediate size parameters. This proposal is further supported by the sum rule for integrated extinction.

1. Introduction

The literature on light scattering by aspherical particles is vast, e.g., [1], ranging from radiative transfer, climatology and remote sensing of atmospheric aerosols [2,3] and microscopy of bacteria [4] to astrophysics of interstellar dust [5] and marine monitoring [6]. Any bounds that can be set on optimal shapes, not only provide insight but can also be of great utility. For example, in the geometrical optics limit, relevant to optically large particles, twice the geometric cross-section is a good approximation to the total extinction cross-section. Combined with a remarkable theorem, due to Cauchy, that orientation-averaged cross-sectional area of an ovaloid equals one-quarter of its surface area, the geometrical limit implies that spherical total cross-sections are always lower than those for any randomly oriented convex particles of equal volume. While perhaps not widely appreciated, this approximation was discussed in important papers in optics and atmospheric science [7,8]. The effect can be illustrated by considering spheroidal surface area, normalized by that of an equal volume sphere (denoted S_r), regarded as a function of the aspect ratio, e.g., see p. 620 of [9], given by $S_r=(1/2)(1-e^2)^{-1/3}+(1/4e)(1-e^2)^{2/3}\ln[(1+e)/(1-e)]$ where $e^2\equiv 1-(c/a)^2$ for oblate spheroids, and $S_r=(1/2)(1-e^2)^{1/3}+(1/2e)(1-e^2)^{-1/6}\sin^{-1}(e)$ where $e^2\equiv 1-(b/a)^2$ for prolate spheroids. The function, plotted in Fig. 1 vs. the aspect ratio ρ (curve labelled “geometric”), has a minimum at the spherical value of $\rho=1$. Note that the validity of the geometric limit, because of the optical theorem, is rather broader than might at first be expected [7].

The success of a simple geometric reasoning in the large particle limit prompted us to ask an analogous question for the small particle (low frequency) Rayleigh scattering regime. Here the physics of scattering is entirely different: governed by the magnitude of the induced

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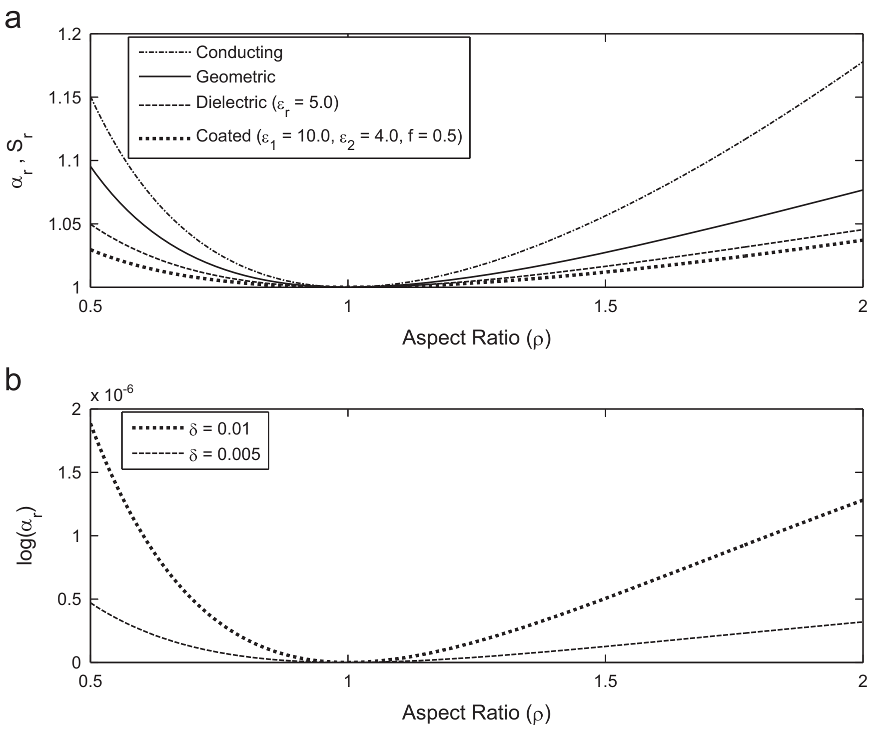

Fig. 1. Optimality of the spherical shape: polarizabilities. Top panel (a): Relative (normalized by the equal-volume-sphere value) surface area S_r (solid line), relative orientation-averaged polarizability (α_r) of conducting spheroids (dashed-dotted line), α_r of dielectric ellipsoids (dashed line), and α_r of confocal coated ellipsoid (dotted) vs. ρ, the aspect ratio. Bottom panel (b): Relative orientation-averaged polarizability of dielectric ellipsoids for two different values of the dielectric contrast δ (see text) in the Rayleigh–Gans (transparent) regime. The ordinate is the logarithm of the spherical access (see Eq. (11) in text). For both panels, oblate/prolate spheroids on left/right of unity.

dipole moment. Yet, the classical picture of a dielectric sphere placed in a uniform external electric field, resulting in the displaced net positive and negative charges on the opposing surfaces, evokes geometrical reasoning. Hence, we ask: do convex aspherical particles scatter more strongly than equivalent volume spheres? To render the question well posed, we further specify that the particles are randomly oriented and consider the magnitude of the orientation-averaged induced dipole moment. Thus, the question, apparently not raised before in the context of scattering theory, becomes: do randomly oriented convex aspherical particles (ovaloids) possess orientation-averaged magnitude of an induced dipole moment larger than that of equivalent volume spheres? By a way of preview, the answer is in the affirmative for a wide variety of circumstances. However, having conducted an extensive literature survey of related questions about particle shapes, we frequently encountered conflicting statements, scattered across a variety of disciplines.

For example, early influential developments in electromagnetics included statements such as one by Siegel [10, p. 294], *When the wavelength is much longer than the dimensions of a body, one cannot discern details of the structure of the body: the observed effect depends more on the size of the body than on its shape.* It is implied that, in the long wavelength regime, particle shape is not essential. Consider, for example, thermal IR remote sensing studies of aerosols which commonly ignore asphericity and model the scatterers by size-distributed Mie spheres, e.g., see pp. 1213–1214, Section 3.2, and data in Fig.11 of [11]. This is not a criticism as spherical modelling suffices for many purposes and authors in the field of atmospheric optics are well aware of the importance of asphericity, e.g., [2,3,12–16]. Yet, integral statements of shape optimality are rare in the small particle regime. Consider, for instance, an important and insightful recent review in this journal [17] where in the 2nd paragraph of the Abstract it is stated that "*for particles much smaller than the wavelength of incident light, absorption is proportional to the particle volume and mass*". This statement must be qualified by specifying particle shape. Otherwise, as discussed below, spheroids absorb and scatter more than equal-volume (mass) spheres.

Early literature in radar meteorology dealt with shape effects but only for the case of backscatter, e.g., [18], concluding numerical calculations with the conjecture that spheroids do have larger echoes than equal volume spheres. In applied optics, the question of optimal shape for absorption in the visible was tackled almost at the same time by Senior and by Bohren and Huffman [19,20], arriving at seemingly conflicting results and ascribing the spherical shape the minimal vs. the maximal absorption cross-section status, respectively. As was argued in [20], the discrepancy had to do with the chosen values of the dielectric constant. Polarizability has also been studied in material science where the emphasis is placed on effective properties of materials and mixture rules rather than on scattering. We note, in particular, a series of studies by Sihvola and colleagues [21–23], computing scalar